

CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	204	ALL (except EC)
Examination	Date	Pages
Final	April 2016	2
Instructor	Course Examiners	
All	E. Cohen	

Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

1. Using Gauss-Jordan method, find all solutions of the following system of equations:

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 5 \\ x_1 + 4x_2 - 3x_3 - 3x_4 &= 6 \\ 2x_1 + 3x_2 - x_3 + 4x_4 &= 8 \end{aligned}$$

2. Let $M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

(a) Find M^{-1} .

(b) Calculate the matrix C so that $MC = B$, where $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$.

3. (a) Use Cramer's rule to solve the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 2 \\ x_1 + x_3 &= 3 \\ x_1 + x_2 - x_3 &= 1 \end{aligned}$$

(b) Evaluate the determinant of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 0 & 1 & 4 \\ -1 & 0 & 1 & 3 \\ 2 & 1 & 1 & 4 \end{pmatrix}$

4. Let \mathcal{L} be the line with parametric equations $x = 2 - t$, $y = 1 + t$, $z = 4 + 2t$ and let $v = (1, 1, 1)$. Find vectors w_1 , w_2 such that $v = w_1 + w_2$, and such that w_1 is parallel to \mathcal{L} and w_2 is perpendicular to \mathcal{L} .

5. Let $P_1(0, 1, 1)$, $P_2(1, 1, 0)$, and $P_3(1, 0, 1)$ be 3 points
- Find the area of a triangle with vertices P_1 , P_2 , P_3 .
 - Find the equation of the plane containing P_1 , P_2 , P_3 .
6. Let \mathcal{L} be the line with parametric equations $x = 3 - t$, $y = -1 + 2t$, $z = 1 - 4t$ and let \mathcal{P} be the plane $4x - 2y - 2z = 6$
- Prove that \mathcal{L} and \mathcal{P} are parallel.
 - Find the distance between \mathcal{L} and \mathcal{P} .
7. Let $v_1 = (1, 2, 3)$ and $v_2 = (-1, 2, -1)$
- Find scalars a and b such that $av_1 + bv_2 = (0, 4, 2)$.
 - Find a vector v_3 such that v_1, v_2, v_3 is a basis of \mathbb{R}^3 .

8. Let $A = \begin{pmatrix} 1 & 1 & 0 & 4 & 0 & 6 \\ 0 & 0 & 1 & 2 & 0 & 7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \\ u \\ v \\ w \end{pmatrix}$. Find a basis for the solution space of the homogeneous system $AX = 0$.

9. Let $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$. Find all eigenvalues of A .

Is A diagonalizable? If yes, find P so that $P^{-1}AP = D$ diagonal.

10. Let $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$. Find A^{100} .