# Electric Circuits 1

## ECSE-200 Section: 1

**23 April 2013, 9:00AM**

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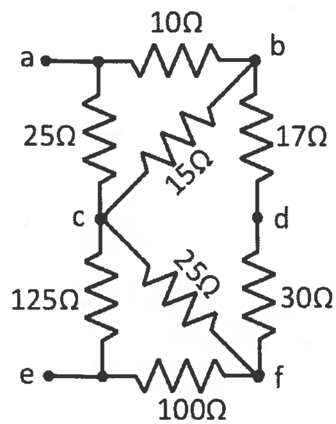
INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **NO CRIB SHEETS** are permitted.
- Provide your answers in an **EXAM BOOKLET**.
- **STANDARD CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, with a total of 6 pages, including the cover page.
- This examination is **PRINTED ON BOTH SIDES** of the paper

1. Consider the circuit below. Answer the questions. [12 pts]



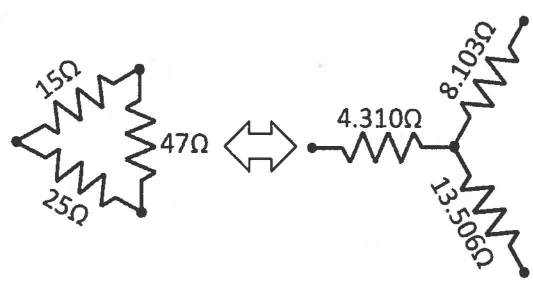

a) What is the definition of a passive element? [1pt]

b) What is the definition of a linear element? [1pt]

c) What is the equivalent resistance between terminals c and d ? [3pts]

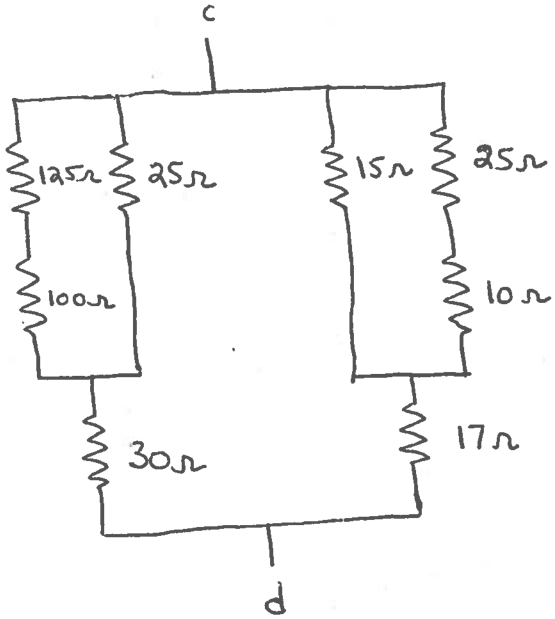
d) What is the equivalent resistance between terminals a and b ? [3pts]

e) What is the equivalent resistance between terminals a and e ? [4pts]
**HINT:** You may find it useful to use the Δ-to-Y transformation below.




a) An element that cannot deliver more energy to a circuit than it has received. [+1]

b) An element where terminal voltage and current are related by a linear function or linear operator. [+1] c)

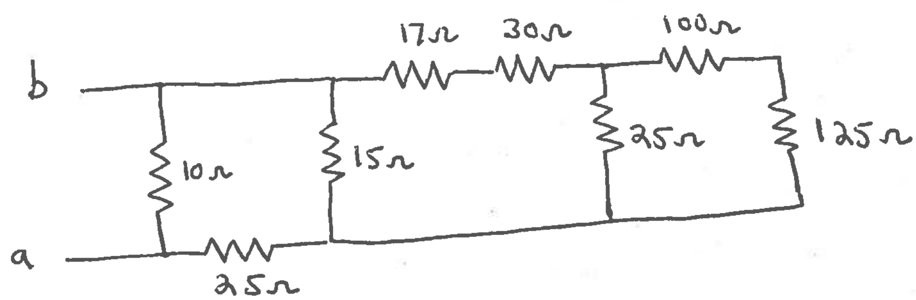



$$R_{cd} = (30\Omega + 225\Omega // 25\Omega) // (17\Omega + 35\Omega // 15\Omega) \quad [+2]$$

$$= 52.5\Omega // 27.5\Omega$$

$$= 18.05\Omega \quad [+1]$$

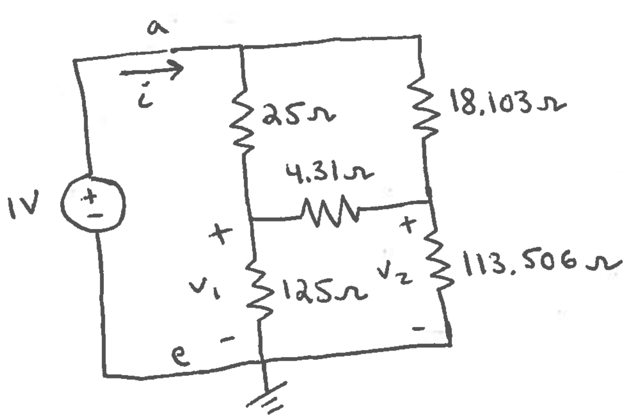
d)




$$R_{ba} = (((225\Omega // 25\Omega + 47\Omega) // 15\Omega) + 25\Omega) // 10\Omega \quad [+2]$$

$$= 7.888\Omega \quad [+1]$$

e)




[+1] for equivalent circuit

[+1] for applying source

$$0 = \frac{v_1}{125} + \frac{v_1 - v_2}{4.31} + \frac{v_1 - 1}{25}$$

$$0 = \frac{v_2}{113.506} + \frac{v_2 - v_1}{4.31} + \frac{v_2 - 1}{18.103}$$

$$\left.\begin{aligned} 0.04 &= 0.2800\, v_1 - 0.232\, v_2 \\ 0.05524 &= -0.232\, v_1 + 0.2961\, v_2 \end{aligned}\right\}$$

$$v_1 = \frac{\begin{vmatrix} 0.04 & -0.232 \\ 0.05524 & 0.2961 \end{vmatrix}}{\begin{vmatrix} 0.2800 & -0.232 \\ -0.232 & 0.2961 \end{vmatrix}} = 0.848\ \text{V}$$

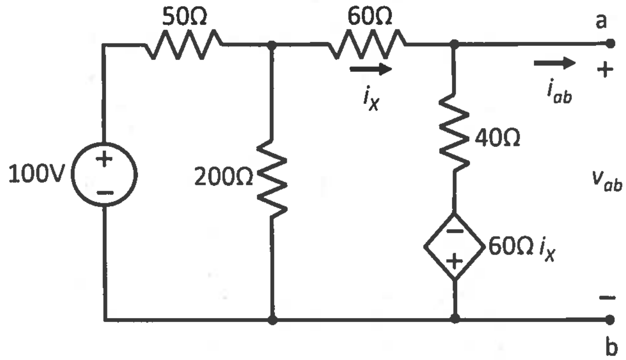
$$v_2 = \frac{\begin{vmatrix} 0.2800 & 0.04 \\ -0.232 & 0.05524 \end{vmatrix}}{\begin{vmatrix} 0.2800 & -0.232 \\ -0.232 & 0.2961 \end{vmatrix}} = 0.851\ \text{V}$$

$$i = \frac{v_1}{125\Omega} + \frac{v_2}{113.506\Omega} = 14.28\ \text{mA}$$

$$R_{ae} = \frac{1\text{V}}{i} = 70.0\ \Omega \quad [+1]$$

[+1]

2. Consider the circuit below. Answer the questions. [12 pts]




a) What is Thévenin's theorem? [1pt]

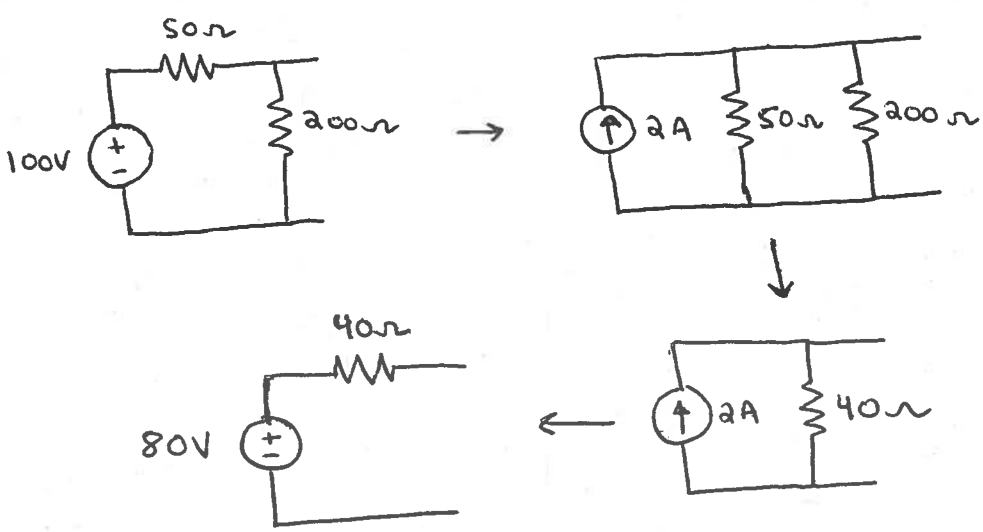
b) Draw the Thévenin equivalent circuit with respect to terminals a and b. Be sure to label the terminals a and b in your diagram. [6pts]

c) What is the maximum power that can be delivered to an optimally chosen load resistor attached to the terminals a and b? [2pts]

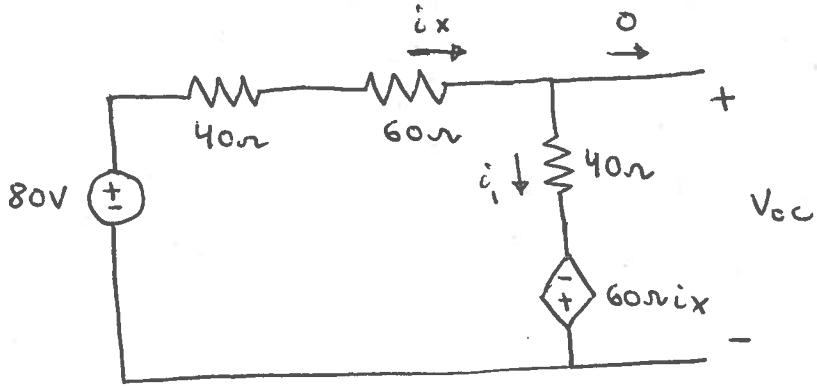
d) A load resistor $R$ is attached to the terminals a and b. What are the two values of $R$ that will cause a power of 1.5W to be absorbed by $R$? [3pts]

a) Any circuit composed of ideal resistors, dependent sources and independent sources is equivalent to a Thévenin circuit (voltage source in series with a resistor). [+1]

b) First simplify circuit:



Find $V_{oc}$. [+1]




KCL: $i_1 = i_x$

KVL: $0 = -80V + 40\Omega i_x + 60\Omega i_x + 40\Omega i_x - 60\Omega i_x$

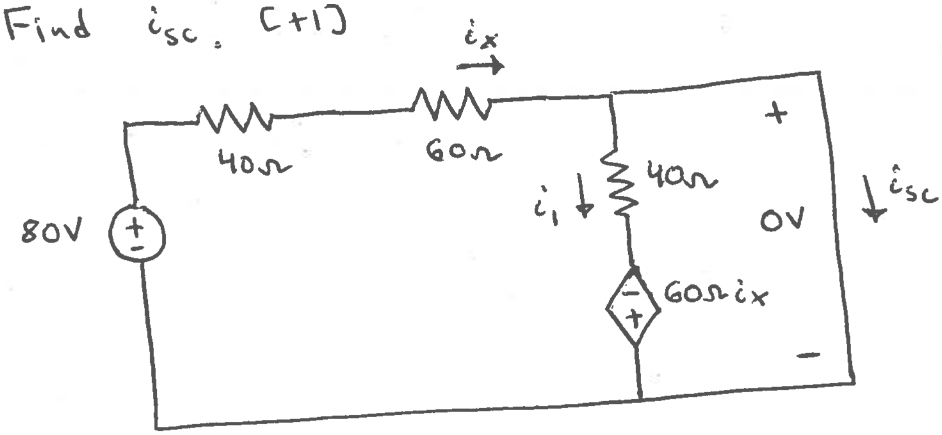
$80V = 80\Omega \cdot i_x$

$i_x = 1A$

KVL: $0 = 60\Omega i_x - 40\Omega i_x + V_{oc}$

$V_{oc} = -20\Omega i_x = -20V$ [+1]

Find $i_{sc}$. [+1]




KVL: $0 = -80V + 40\Omega i_x + 60\Omega i_x$

$i_x = \frac{80V}{100\Omega} = 0.8A$

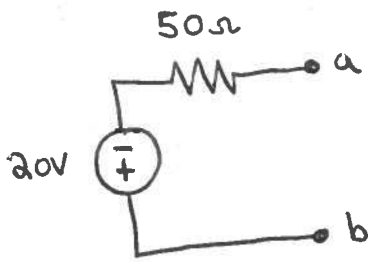
KVL: $0 = 60\Omega \cdot i_x - 40\Omega i_1$

$i_1 = \frac{60\Omega \cdot i_x}{40\Omega} = 1.2A$

KCL: $i_{sc} = i_x - i_1 = -0.4A$ [+1]

$$R_T = \frac{V_{oc}}{i_{sc}} = 50\Omega$$

[+1]

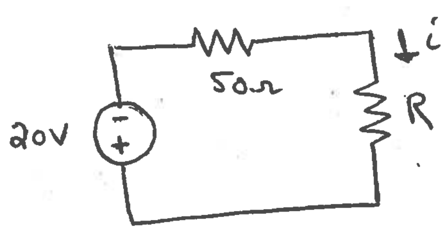



[+1] for diagram

c) $$P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2}$$ [+1]

$$= 2W$$ [+1]

d)




$$P_{abs} = i^2 \cdot R$$

$$1.5W = \left(\frac{20V}{R + 50\Omega}\right)^2 \cdot R$$ [+1]

$$1.5(R+50)^2 = 400R$$

$$1.5R^2 + 150R + 3750 = 400R$$

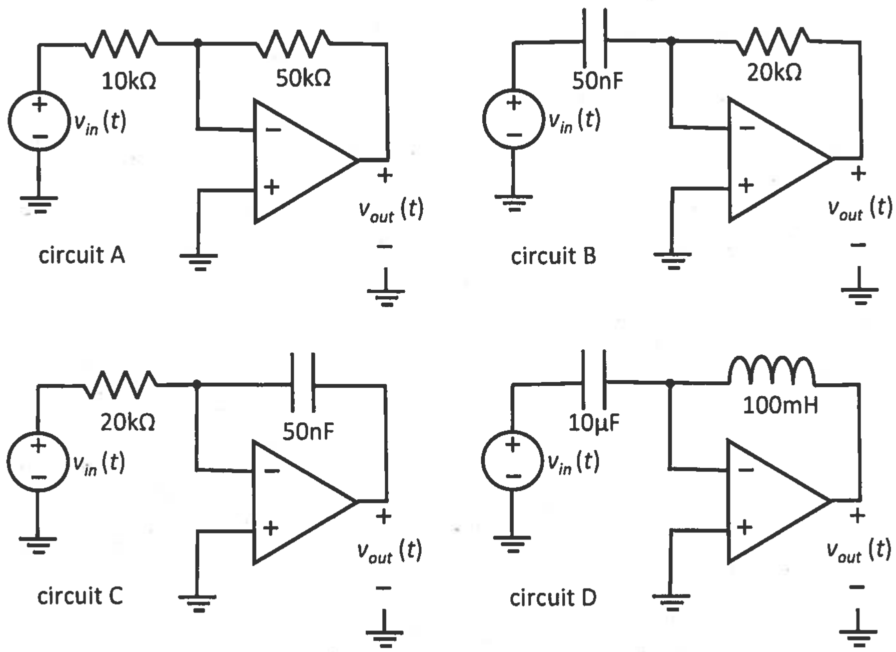
$$1.5R^2 - 250R + 3750 = 0$$

$$R = \frac{250 \pm \sqrt{250^2 - 4 \cdot 1.5 \cdot 3750}}{2 \cdot 1.5}$$

$$= 150\Omega \quad \text{and} \quad 16.67\Omega$$

[+1] [+1]

3. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]




a) Give one reason why negative feedback is used in op-amp circuits. [2pts]

b) How does the output voltage $v_{out}(t)$ depend upon the input voltage $v_{in}(t)$ for circuit A? [2pts]

c) How does the output voltage $v_{out}(t)$ depend upon the input voltage $v_{in}(t)$ for circuit B? [2pts]

d) How does the output voltage $v_{out}(t)$ depend upon the input voltage $v_{in}(t)$ for circuit C? Assume that the capacitor stores zero energy at $t$ = 0s, and consider only $t \geq 0$. [2pts]

e) How does the output voltage $v_{out}(t)$ depend upon the input voltage $v_{in}(t)$ for circuit D? [2pts]

f) Voltage sources of +10V and −10V are used to power the op-amp circuit A. What is the range of input voltages that can be used without causing the op-amp to saturate? [2pts]

a) programmable gain

gain independent of open-loop gain

stable output

any one [+2]

b) $$0 = \frac{0 - v_{in}}{10k\Omega} + \frac{0 - V_{out}}{50k\Omega} \quad [+1]$$

$$V_{out} = -5\, V_{in} \quad [+1]$$

c) $$0 = 50nF \frac{d}{dt}(0 - v_{in}) + \frac{0 - V_{out}}{20k\Omega} \quad [+1]$$

$$V_{out} = -1ms \cdot \frac{dv_{in}}{dt} \quad [+1]$$

d) $$0 = \frac{0 - v_{in}}{20k\Omega} + 50nF \frac{d}{dt}(0 - V_{out}) \quad [+1]$$

$$V_{out} = \frac{-1}{1ms} \int_0^t V_{in}\, dt' \quad [+1]$$

$$t \geq 0$$

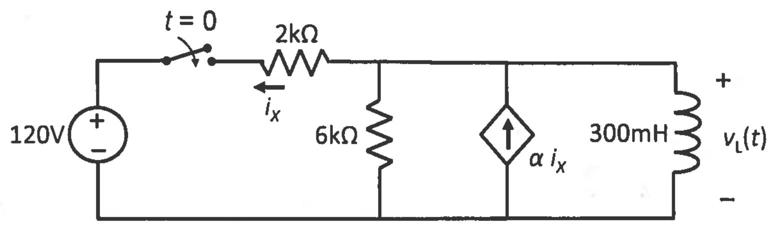
f) $$-10V < V_{out} < +10V \quad [+1]$$

$$\therefore \quad -2V < V_{in} < +2V \quad [+1]$$

e) $$0 = 10\mu F \cdot \frac{d}{dt}(0 - v_{in}) + \frac{1}{100mH} \int_0^t (0 - V_{out})\, dt' \quad [+1]$$

$$V_{out} = -(1ms)^2 \cdot \frac{d^2 v_{in}}{dt^2} \quad [+1]$$

4. Consider the circuit below (useful for firing sparks). The switch is open for $t < 0$s, and closes instantaneously at $t = 0$s. Assume dc steady state behaviour for $t < 0$. Answer the questions. [12 pts]




a) What is the definition of a passive element? [1pt]

b) Assume that $\alpha = 0$. What is the voltage $v_L(t)$ for $t > 0$? Plot your solution for $v_L(t)$ versus $t$. [6pts]

c) What is the value of $\alpha$ that causes the Thévenin resistance with respect to the inductor terminals to become $-3\text{k}\Omega$ for $t > 0$? [2pts]

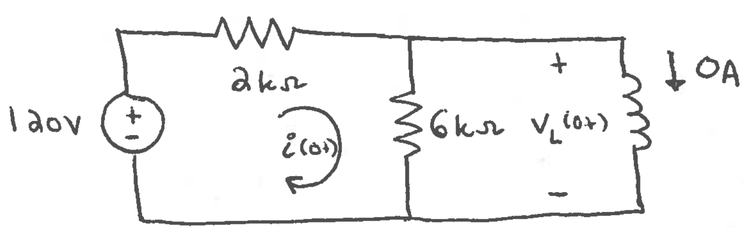
d) For the value of $\alpha$ found in part c), at what time $t$ will $v_L(t) = 36\text{kV}$ ? [3pts]

a) An element that cannot deliver more energy to a circuit than it has received. [+1]

b) Assume $\alpha = 0$.

$t \leq 0$ $\quad i_L(0) = 0\ \text{A}$. [+1]

$t = 0+$




$$i(0+) = \frac{120\text{V}}{2\text{k}\Omega + 6\text{k}\Omega} = 15\text{mA}$$

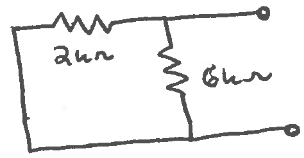
$$V_L(0+) = 6\text{k}\Omega \cdot i(0+)$$

$$= 90\ \text{V} \quad [+1]$$

$t \to \infty$

dc steady state is reached, thus $V_L(\infty) = 0\text{V}$ [+1]

$$\tau = \frac{L}{R_{th}}$$




$$R_{th} = 2k\Omega \parallel 6k\Omega$$
$$= 1.5k\Omega$$

$$= \frac{300mH}{1.5k\Omega}$$

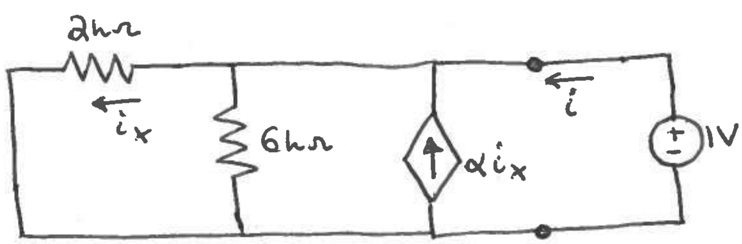
$$= 200\mu s \quad [+1]$$

$$V_L(t) = V_L(\infty) + \left(V_L(0^+) - V_L(\infty)\right)\exp(-t/\tau) \quad [+1]$$

$$= 90V \exp(-t/200\mu s) \quad [+1]$$

$$t > 0$$

c) Find $R_{th}$ in terms of $\alpha$:




$$0 = \frac{1V}{2k\Omega} + \frac{1V}{6k\Omega} - \alpha i_x - i$$

$$i_x = \frac{1V}{2k\Omega} = 0.5mA$$

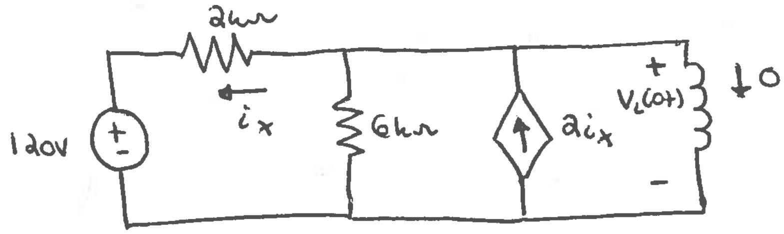
$$i = 1V \cdot \left(\frac{1}{2k\Omega} + \frac{1}{6k\Omega} - \frac{\alpha}{2k\Omega}\right)$$

$$R_{th} = \frac{1V}{i} = -3k\Omega \quad [+1] \qquad \left(\text{or } R_{th} = \frac{V_{oc}}{i_{sc}} = -3k\Omega \ [+1]\right)$$

$$\frac{1V}{1V \cdot \left( \frac{1}{2k\Omega} + \frac{1}{6k\Omega} - \frac{\alpha}{2k\Omega} \right)} = -3k\Omega$$

$$\alpha = 2 \quad [+1]$$

d) $t = 0+$




$$0 = \underbrace{\frac{V_L(0+) - 120V}{2k\Omega}}_{i_x} + \frac{V_L(0+)}{6k\Omega} - 2 \cdot \left( \frac{V_L(0+) - 120V}{2k\Omega} \right) + 0$$

$$V_L(0+) = \frac{120V/2k\Omega}{1/2k\Omega - 1/6k\Omega} = 180V$$

$t \to \infty$ dc steady state would be reached, $V_L(\infty) = 0V$

$$\tau = \frac{L}{R_{th}} = \frac{300mH}{-3k\Omega} = -100\mu s$$

$$V_L(t) = V_L(\infty) + (V_L(0+) - V_L(\infty)) \exp(-t/\tau)$$

$$= 180V \exp(t/100\mu s) \quad [+1]$$

$$36\ kV = 180V \exp(t/100\mu s)$$

$$t = 100\mu s \cdot \ln\left(\frac{36000}{180}\right)$$

$$= 530\ \mu s \quad [+1]$$