Assignment #3
Transportation Problem, Integer/Binary Programming and Goal Programming

ADM2302 students are reminded that submitted assignments must be neat, readable, and well-organized. Assignment marks will be adjusted for sloppiness, poor grammar and spelling, as well as for technical errors. While working together is encouraged, plagiarism on assignments will not be accepted. The assignment is to be submitted electronically as a **single PDF file** via blackboard learn by Sunday November 27th prior to 23:59. Front page of the PDF document has to include title of the assignment, course code and section, student name and student number. Second page is *the individual statement of integrity that must be signed.*

Note: *Each student must provide an individual original submission of completed Assignment #3.* Please also note: Assignment #3 copies that are submitted jointly (i.e., by more than one author) will not be graded.

This assignment cannot be hand written. Solutions to all exercises in this assignment are to include "managerial statements" that communicate the results of the analyses.

E-mail questions related to the assignment should be sent to the Teaching Assistant or posted on the Blackboard Learn course website "Discussion Area" (viewed by all).

Total Value = 70 points

General marking rules

- After inputting the final grade on the 1st page, if the statement of integrity is not available or not signed by a given student then deduct 7 points. On the front page, show the "original grade" – 7 = "new grade".

- Don't penalize twice for an error that occurs at the start and it does affect the results that follows.

- Provide a brief explanation that would allow the students to understand were the error was committed and know what the right solution is.

- **Please provide me with the most common mistakes, so I can provide feedback to the students and go over the concepts in class.**

Problem 1 (26 points)

The J. Mehta Company's production manager is planning a series of one-month production periods for stainless steel sinks. The forecasted demand for the next four months is as follows:

Month	Demand for Stainless Steel Sinks
1	120
2	160
3	240
4	100

The Mehta firm can normally produce 100 stainless steel sinks in a month. This is done during regular production hours at a cost of $100 per sink. If demand in any one month cannot be satisfied by regular production, the production manager has three other choices:

(1) he can produce up to 50 more sinks per month in overtime but at a cost of $130 per sink;
(2) he can purchase a limited number of sinks from a friendly competitor for resale (the maximum number of outside purchases **over the four-month period** is 450 sinks, at a cost of $150 each);
(3) Or, he can fill the demand from his on-hand inventory (i.e. beginning inventory). The inventory carrying cost is $10 per sink per month (i.e. the cost of holding a sink in inventory at the end of the month is $10 per sink).

A constant workforce level is expected. Back orders are NOT permitted (e.g. order taken in period 3 to satisfy the demand in later period 2 is not permitted). Inventory on hand at the beginning of month 1 is 40 sinks (i.e. beginning inventory at month 1 is 40 sinks)

a. Set up and formulate algebraically the above "production scheduling" problem as a **TRANSPORTATION Model** to minimize cost. (16 points)

b. SOLVE using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Also include a managerial statement that describes verbally the results. (10 points)

Note: This problem can be formulated as multi-period production scheduling LP problem. However, **if you try to formulate it this way then you will get ZERO as the problem requirement is to formulate it as a transportation problem.**

SOLUTIONS

Mehta's production smoothing problem is a good exercise in the formulation of transportation problems and applying them to real-world issues.

This problem is a multipored scheduling production LP problem. **If the student tried to formulate it this way then ZERO mark will be given to the student as I asked for the problem to be formulated as a transportation problem.**

Please note that we did not ask the student to draw the transportation network of the problem thus no mark will be allocated for such a network…However we encouraged them to draw the network to help them figure out the LP formulation.

In applying a transportation model excel spreadsheet formulation to solve such a problem in using solver, a very large cost (say about $5,000) would be assigned to each cell where backorder is not allowed. This would assure that they would not appear in the final solution (another alternative is to write down a constraint that set these cell equal to zero).

a. Model Formulation **(16 points)**

Let

BIj = Beginning inventory at the start of month 1 to meet demand of month
Xij = the number stainless sinks manufactured through i to meet demand of month j
Where i = r1, o1, r2, o2, r3, o3, r4, o4, and s
j = 1, 2, 3, 4 **(2 points)**

Min Z = $100(xr11+xr22+xr33+xr44) + 110(xr12+xr23+xr34) + 120(xr13+xr24) +130xr14 + 130(xo11+xo22+xo33 +xo44) + 140(xo12 + xo23+xo34) + 150(xo13+xo24) + 160xo14 + 150(xs1 + xs2 + xs3 + xs4) + $10BI2 + $20BI3 + $30BI4 **(3 points)**

Constraints = 11 points (do NOT deduct point if the student does not provide a description for the constraints (i.e. supply constraints, demand constraints, etc.)

Subject to
Supply Constraints (6 points)
BI1 + BI2 +BI3 + BI4 <= 40 (1 point)

xr11+xr12+xr13 + xr14 <= 100 (1 point)
xo11+xo12+xo13 + xo14<= 50

xr22+xr23+xr24 <= 100 (1 point)
xo22+xo23+xo24<= 50

xr33+xr34 <= 100 (1 point)
xo33+xo34 <= 50

Xr44 <= 100 (1 point)
Xo44 <= 50

xs1 + xs2 + xs3 + xs4<= 450 (1 point)

Demand Constraints **(4 points)**

BI1+ xr11 + xo11 + xs1 = 120
BI2 + xr12 + xo12 + xr22 + xo22 + xs2 = 160
BI3 + xr13 + xo13 + xr23 + xo23 + xr33 + xo33 + xs3 = 240
BI4 + xr14 + xo14 + xr24 + xo24 + xr34 + xo34 + xr44 + xo44 + xs4 = 100

all variables >= 0 **(1 point)**

Please read carefully the following:

If they use >= for the demand constraints instead of "=" and given that the supply constraints are <= then it is also correct since it is a minimization problem.

If the left hand side of the constraint and the right hand of the constraint is correct for ALL constraints above but the equality sign is different then follow this marking scheme:

All constraints = (Infeasible solution): **deduct 3 points**

All constraints <= : **deduct 3 points**

Supply constraints = and demand constraints >= **deduct 3 points**

b. Excel Solution **(7 points)**

Refer to the Excel file
THE STUDENTS HAVE TO SET IT the EXCEL SPREADSHEET FORMULATION SIMILAR TO THE ONE PROVIDE (i.e. two tables: the transportation parameter table and solution table). IF THEY DON'T THEN DEDUCT **4 POINTS**)

In applying a transportation model excel spreadsheet formulation to solve such a problem in using solver, a very large cost (say about $5,000) would be assigned to each cell where backorder is not allowed. This would assure that they would not appear in the final solution (another alternative is to write down a constraint that set these cell equal to zero).

Please note that this problem have multiple optimal solution (check the sensitivity report).
(3 points): **Managerial Statement**
The optimal minimum cost for the production plan is $ 65,300.

Optimal Production Plan 1

- Manufacture at regular time 100 units in months 1, 2, 3, and 4, respectively, as well as 20 units of overtime in month 1, and 50 units at over-time in month 2, 3 and 4.
- Subcontract 60 units.

Optimal Production Plan 2:

- Manufacture at regular time 100 units in months 1, 2, 3, and 4, respectively, as well as 50 units at over-time in month 1, 2, 3 and 4.
- Subcontract 30 units.

Problem 2 (21 points)

A department has three machines available, and the new department manager must select one of the machines to assign to a new product line. The product line will consist of three slightly different products, A, B and C. Production requirements, machine capacities and setup costs are given in the following table:

Machine	**Setup cost**	**Production time per pound**			**Capacity (hours)**
		A	**B**	**C**	
1	$150	4	3	5	1000
2	$120	3	2	4	800
3	$110	2	4	2	700

The revenue per pound on the three products is listed in the following table:

Product	**Revenue**
A	$13
B	$10
C	$12

a. Formulate algebraically this problem that will maximize the net profit taking into account an additional factor: At least 40 pounds of each product must be made. (14 points)

b. Solve for the optimal solution and profit using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Also include a managerial statement that describes verbally the results. (7 points)

SOLUTIONS

a. Algebraic formulation **(14 points)**

Xij = pounds of product i produced on Machine j **(1 point)**
Where i = A, B, C and j = 1, 2, 3

Yj = 1 if Machine j is chosen, 0 otherwise **(1 point)**
Where j = 1, 2, 3

Maximize Z = 13 (XA1 + XA2 + XA3) + 10(XB1+XB2+XB3) + 12(XC1+XC2+XC3) – 150 Y1 – 120Y2 – 110Y3 **(2 points)**

Subject to
(3 points for the set of three constraints below, if the Yi are not included in these constraints then the student gets ZERO)
4XA1 + 3XB1+5XC1 <= 1000 Y1
3XA2 + 2XB2+4XC2 <= 800 Y2
2XA3 + 4XB3+2XC3 <= 700 Y3

Y1+Y2+ Y3 = 1 **(2 points)**

(3 points for the set of three constraints below)
XA1 + XA2+ XA3 >= 40 (given the above constraints two decision variables out of the three would be equal to Zero)
XB1 + XB2+ XB3 >= 40
XC1 + XC2+ XC3 >= 40

Y1, Y2, Y3 = binary (if student defined decision variables as binary then no need to write this but if they write it is still ok).
Xij >=0 **(1 point)** and Integer **(1 point)**
for all i= A,B, C and j = 1, 2, 3

Excel Solver Spreadsheet formulation (5 points)

	XA1	XB1	XC1	XA2	XB2	XC2	XA3	XB3	XC3	Y1	Y2	Y3			
	0	0	0	0	0	0	230	40	40	0	0	1			
One machine	0	0	0	0	0	0	0	0	0	1	1	1	1	=	1
Capacity M1	4	3	5	0	0	0	0	0	0	-1000	0	0	0	<=	0
Capacity M2	0	0	0	3	2	4	0	0	0	0	-800	0	0	<=	0
Capacity M3	0	0	0	0	0	0	2	4	2	0	0	-700	0	<=	0
Min A	1	0	0	1	0	0	1	0	0	0	0	0	230	>=	40
Min B	0	1	0	0	1	0	0	1	0	0	0	0	40	>=	40
Min C	0	0	1	0	0	1	0	0	1	0	0	0	40	>=	40
Profit/Cost	13	10	12	13	10	12	13	10	12	-150	-120	-110			
Solution	0	0	0	0	0	0	2990	400	480	0	0	-110		Total	
														$3,760.00	

Managerial statement (2 points): Produce on Machine 3, 230 units of product A, 40 units of product B and C. The optimal profit is $3,760.

Problem 3 (23 points)
California Tours is planning a group bus trip that for the following candidate cities in the table below. Also included are the total costs for the group to visit those cities on the tour.

City	Cost ($)
San Francisco (SF)	5000
Oakland (OK)	4500
Palo Alta (PA)	3600
San Jose (SJ)	4100
San Mateo (SM)	3500
Concord (CO)	2500
Santa Cruz (SC)	3200
Monterey (MN)	4000

To plan the trip, three prioritized goals are listed below, in order of importance.

P1: Avoid spending more than $15000 for the total trip.
P2: Visit at least 5 cities.
P3: Include San Mateo in the tour.

a. Formulate a goal programming model that will help to determine the number of cities to include in the tour. (14 points)

b. Find the optimal solution using Solver. (Provide a printout of the Answer report and the Excel spreadsheet formulation). Also include a managerial statement that describes verbally the results. (9 points)

SOLUTIONS

a. Algebraic Formulation (14 points)

Let Xi = 1 if bus stops at city i
0 otherwise
where i= 1(SF), 2(OK), 3(PA), …, 8(MN) **(2 points)**

dj-: underachievement of goal j (2 points)
dj+: overachievement of goal j
where j = 1, 2, 3
(note that d3+ is not possible since X5 is binary)

Min P1(d1+) + P2(d2-) + P3[(d3-)+(d3+)] or Min P1(d1+) + P2(d2-) + P3(d3-) (3 points)
OR
Min P1(d1+), P2(d2-), P3[(d3-)+(d3+)] or Min P1(d1+), P2(d2-), P3(d3-)

Subject to

$5000X1+4500X2+…+8000X8 + (d1-) – (d1+) = 15,000 (2 points)

X1+X2+X3+X4+X5+X6+X7+X8 + (d2-) – (d2+) = 5 (2 points)

X5 + (d3-) = 1 OR X5 + (d3-) – (d3+) = 1 (because X5 is a binary variable then both correct) (2 points)

Xi = 0 or 1 for all i= 1, 2, …,8
di-, di+ >= 0 (1 point)

b. Find the optimal solution using Solver. (Provide a printout of the Answer report and/or the Excel spreadsheet formulation**). (9 points: 6 points for solving and 3 points for managerial statement)**

The student needs to solve THREE linear programming problems on Excel Solver.
If the solve it as a weighted Goal programming DEDUCT 4 points.

First LP Problem (2 points); Objective function: Min d1+.
Solution: Goal 1 is satisfied with d1+ = 0.

Thus we did not spend more than $15,000 (managerial statement) (1 point)

Second LP Problem (2 points); Objective function: Min d2-
And add a constraint: d1+ = 0
Solution: Goal 2 is NOT satisfied with d2- = 1.
Thus we manage to satisfy the visit of four cities instead of five cities (managerial statement) (1 point)

Third LP Problem (2 points):
Objective function: Min d3- or Min (d3-) + (d3+)
And add a constraint d2- = 1
Solution: Goal 3 is satisfied with d3- = 0.
Thus San Mateo is included in the tour (managerial statement) (1 point)

Note students can have an overall managerial statement (which is equivalent to the above separate one): example - We will spend $14,300 to visit 4 cities that include Palo Alto, San Mateo, Santa Cruz and Monterey. We could not satisfy the goal of visiting 5 cities as for the other two goals they were satisfied.

Source

Problem 1: B. Render, R.M. Stair, Jr., N. Balakrishnan, and B. Smith 2010. *Managerial Decision modeling with spreadsheets*, 2nd Canadian edition. Pearson Education Canada, Toronto, Ontario.

Problem 2: Stevenson, W.J. and Ozgur, C. 2007. Introduction to Management Science with Spreadsheets. McGraw-Hill/Irwin. Boston, Mass. 812p.