

**Department of Computer Science and Software Engineering  
Concordia University  
COMP 335: Introduction to Theoretical Computer Science  
Fall 2015**

**Assignment 3  
Evaluation: 50 pts  
(3% of your final grade)**

**Due date and time: Sunday, November 8<sup>th</sup> 2015 by midnight**

**Question 1 (4.5 pts)**

Find context-free grammars for the following languages (with  $n \geq 0, m \geq 0$ )

- (a)  $L = \{a^n b^m : n \neq m - 1\}$
- (b)  $L = \{a^n b^m : n \neq 2m\}$
- (c)  $L = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$

**Question 2 (4 pts)**

Answer the following questions:

- (a) Is it possible for a regular grammar to be ambiguous?
- (b) Show that the language  $L = \{ww^R : w \in \{a, b\}^*\}$  is not inherently ambiguous.

**Question 3 (3.5 pts)**

Show that the following grammar is ambiguous .

$$S \rightarrow aSbS \mid bSaS \mid \lambda$$

**Question 4 (4 pts)**

1. a. Eliminate all the useless productions from the grammar below:

$$\begin{aligned} S &\rightarrow aS \mid AB \\ A &\rightarrow bA \\ B &\rightarrow AA \end{aligned}$$

- b. What language does this grammar generate?

2. Eliminate all the useless productions from the grammar below:

$$\begin{aligned}
S &\rightarrow a|aA|B|C \\
A &\rightarrow aB| \lambda \\
B &\rightarrow Aa \\
C &\rightarrow cCD \\
D &\rightarrow ddd
\end{aligned}$$

**Question 5:** (6 pts)

Answer each part for the following context-free Grammar G

$$\begin{aligned}
R &\rightarrow XRX| S \\
S &\rightarrow aTb| bTa \\
T &\rightarrow XTX| X| \lambda \\
X &\rightarrow a| b
\end{aligned}$$

- |  |   |
|--|---|
| a. What are the variables of G?            | i) True or False: $T \Rightarrow^* T$       |
| b. What are the terminals of G?            | j) True or False: $XXX \Rightarrow^* aba$   |
| c. Which is the start variable of G?       | k) True or False: $X \Rightarrow^* aba$     |
| d. Give three strings in $L(G)$            | l) True or False: $T \Rightarrow^* XX$      |
| e. Give three strings <u>not</u> in $L(G)$ | m) True or False: $X \Rightarrow^* XXX$     |
| f. True or False: $T \Rightarrow^* aba$    | n) True or False: $S \Rightarrow^* \lambda$ |
| g. True or False: $T \Rightarrow^* aba$    | o) Give a description in English of $L(G)$  |
| h. True or False: $T \Rightarrow T$        |   |

**Question 6** (4 pts)

Let the context-free grammar be given whose rules are

$$S \rightarrow SbS | ScS | a.$$

Give all the parse trees that generate string  $abaca$ . Furthermore, give the leftmost derivation corresponding to each of these parse trees.

**Question 7** (6 pts)

1. Answer the following questions for the context-free grammar  $G$  given by the following rules:

$$S \rightarrow aSa | aSb | bSa | bSb | \lambda$$

- (a) Give informal English descriptions of the language  $L(G)$  which the grammar generates.  
(b) The language  $L(G)$  turns out to be a regular language. Give a regular grammar that generates  $L(G)$ .

2. Answer the following questions for the context-free grammar  $G$  given by the following rules:

$$S \rightarrow aS | Sb | a | b |.$$

- (a) Give informal English descriptions of the language  $L(G)$  which the grammar generates.  
(b) Give a regular grammar that generates  $L(G)$ .

### Question 8 (3 pts)

Transform a context-free grammar given by the following rules to an equivalent one in Chomsky normal form

$$\begin{aligned} S &\rightarrow aB \mid bA, \\ A &\rightarrow a \mid aS \mid bbA, \\ B &\rightarrow b \mid bS \mid aBB \end{aligned}$$

### Question 9 (9 pts)

In this exercise refer to the grammar with start symbol **sentence**, set of terminals  $T = \{the, sleepy, happy, tortoise, hare, passes, runs, quickly, slowly\}$ , set of nonterminals  $N = \{\text{noun phrase, transitive verb phrase, intransitive verb phrase, article, adjective, noun, verb, adverb}\}$ , and productions:

**sentence** → **noun phrase transitive verb phrase noun phrase**

**sentence** → **noun phrase intransitive verb phrase**

**noun phrase** → **article adjective noun**

**noun phrase** → **article noun**

**transitive verb phrase** → **transitive verb**

**intransitive verb phrase** → **intransitive verb adverb**

**intransitive verb phrase** → **intransitive verb**

**article** → *the*

**adjective** → *sleepy*

**adjective** → *happy*

**noun** → *tortoise*

**noun** → *hare*

**transitive verb** → *passes*

**intransitive verb** → *runs*

**adverb** → *quickly*

**adverb** → *slowly*

1. Use the set of productions to show that each of these sentences is a valid sentence.
  - a) *the happy hare runs*
  - b) *the sleepy tortoise runs quickly*
  - c) *the tortoise passes the hare*
  - d) *the sleepy hare passes the happy tortoise*
2. Show that *the hare runs the sleepy tortoise* is not a valid sentence.

### Question 10 (6 pts)

Let  $G = (V, T, S, P)$  be the phrase-structure grammar with  $V = \{0, 1, A, B, S\}$ ,  $T = \{0, 1\}$ , and set of productions  $P$  consisting of

$S \rightarrow 0A, S \rightarrow 1A, A \rightarrow 0B, B \rightarrow 1A, B \rightarrow 1.$

- a) Show that 10101 belongs to the language generated by  $G$ .
- b) Show that 10110 does not belong to the language generated by  $G$ .
- c) What is the language generated by  $G$ ?

**Submission:**

- **Assignment must be done individually (no groups are permitted).**
  - Your file should be called *a#\_studentID*, where # is the number of the assignment and *studentID* is your student ID number. For example, for the second assignment, student 123456 would submit a pdf file named a3\_123456.pdf
- Your assignment must be handed either electronically or in the assignments box.
  - a) **Assignments handled electronically** must be **via EAS. Make sure that you upload the assignment to the correct directory of Assignment 3 using EAS. Assignments uploaded to the wrong directory will be discarded and no resubmission will be allowed.** Electronic submission can be in PDF or scans of clear handwriting.
  - b) **Printed handled assignment must be in the assignment Box** (in EV building 3<sup>rd</sup> floor, next to the computer science department main office) under the proper course section E or G).
- Make sure you write the following statement on your assignment:  
*“I certify that this submission is my original work and meets the Faculty's Expectations of Originality”*, with your signature, I.D. #, and the date.

**Note: Assignment not submitted by the due date and in the correct format and/or to the correct location will not be graded – NO EXCEPTIONS!!!!**