

**University of Ottawa**  
**MAT 1332 Practice Midterm Exam**  
**Duration: 80 Minutes.**

Family Name: \_\_\_\_\_

First Name: \_\_\_\_\_

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted**.
- Only the Faculty approved calculators (TI-30X, TI-34X, Casio FX-260X and Casio FX-300X) are allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Student number: \_\_\_\_\_, Total marks: \_\_\_\_\_ out of 30

Problem	1	2	3	4	5	6	7
Marks							

**Question 1.** [4 points] Calculate

a)  $\int_{-5}^5 \frac{14}{y^2 + y - 12} dy$       b)  $\int_1^e \frac{1}{x[1 + (\ln x)^2]} dx$

**Question 2.** [3 points] Solve the differential equation

$$\frac{dy}{dt} = \frac{7t \cos t}{y}$$

with initial condition  $y(0) = 5$ .

**Question 3.** [4 points] Evaluate the integral

$$\int \frac{x^3 + 8x^2 + 5}{x^2 + 5x - 14} dx.$$

**Question 4.** [6 points] For each of the following improper integrals, determine whether it converges, and determine its value if it does.

a)  $\int_1^{13} \frac{1}{t \ln t} dt$       b)  $\int_0^{\infty} \frac{e^{3t}}{1 + e^{6t}} dt$       c)  $\int_1^{\infty} \frac{\ln x}{x^7} dx$

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**Question 5.** [4 points] Consider the functions  $f(x) = 5x^2$  and  $g(x) = -9x + 2$ .

a) Find the area between the two functions.

b) Find the volume of the solid obtained by rotating the area between  $f(x)$  and  $g(x)$  about the  $x$ -axis.

**Question 6.** [4 points] Zombies have invaded campus! Initially, there are 5 zombies. They recruit more of the undead to their ghoulish ranks at rate

$$\frac{dz}{dt} = 8te^{-0.07t},$$

where  $t$  is the time in days and  $z$  are the number of zombies. How many will be infected if the zombies recruited forever?

~~Handwritten work for the problem, including the differential equation and several integration attempts.~~

~~$$\int 8te^{-0.07t}$$

$$\int te^{-0.07t} dt$$

$$u = t \quad v' = e^{-0.07t}$$

$$u' = 1 \quad v = -\frac{e^{-0.07t}}{0.07}$$

$$8 \left[ \frac{-te^{-0.07t}}{0.07} - \int (1) \left( \frac{-e^{-0.07t}}{0.07} \right) dt \right]$$

$$8 \left[ \frac{-te^{-0.07t}}{0.07} + \frac{1}{0.07} \int e^{-0.07t} dt \right]$$

$$8 \left( \frac{1}{0.07} \right) \left( -te^{-0.07t} - \frac{e^{-0.07t}}{0.07} \right) + C$$

$$8 \left( \frac{100}{7} \right) (-e^{-0.07t}) \left( t + \frac{1}{0.07} \right) + C$$

$$\frac{800}{7} (-e^{-0.07t}) \left( t + \frac{100}{7} \right) + C$$

$$\frac{800}{7} (-e^{-0.07t}) \left( \frac{7}{100} \right) \left( \frac{100t}{7} + 1 \right) + C$$~~

~~$$\int z' = \int 8te^{-0.07t}$$

$$= 8 \int te^{-0.07t}$$

$$\int te^{-0.07t}$$

$$\int te^{-\frac{7t}{100}} dt$$

$$u = \frac{t}{100}$$

$$\frac{du}{dt} = \frac{1}{100}$$

$$100 du = dt$$

$$\int 100u(e^{-7u})(100 du)$$

$$\int 10000 ue^{-7u}$$

$$10000 \int ue^{-7u}$$

$$m = u \quad m' = e^{-7u}$$

$$m' = 1 \quad m = -\frac{e^{-7u}}{7}$$

$$10000 \left[ -\frac{ue^{-7u}}{7} - \int -\frac{e^{-7u}}{7} du \right]$$

$$\frac{10000}{7} \left[ -ue^{-7u} - \frac{e^{-7u}}{7} \right] + C$$

$$\frac{(8)(10000)(-e^{-u})(u-1)}{7} + C$$

$$- \frac{80000}{7} \left( \frac{e^{t/100}}{7} \right) \left( t/100 - 1 \right) + C$$

$$- \frac{80000}{49} \left( e^{t/100} \right) \left( \frac{1}{100} \right) (t + 100) + C$$

$$- \frac{800}{49} \left( e^{t/100} \right) (t + 100) + C$$~~

**Question 7.** [5 points] Determine the average value of  $f(x) = \cos x(1 - \sin^2 x)$  over the range  $0 \leq x \leq \frac{\pi}{2}$ .

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