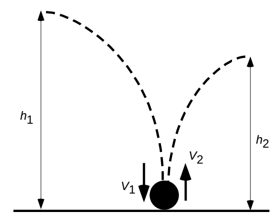
BCEE 231 – Assignment Set #1

P1.1 (30 marks)

A ball is dropped from a helicopter at the height h_1 = 450 m. When it touches the ground, its vertical velocity is $V_1 = \sqrt{2gh_1}$ where g = 9.81 m/s^2. The ball bounces off the ground with the vertical velocity $V_2 = \alpha V_1$ where α = 0.72 is the *coefficient of restitution*. It reaches the height of $h_2 = \frac{V_2^{\ 2}}{2g}$.



Write the computer program that produces the following output:

	h1	V1	V2	h2
Bounce 1	450	…	…	…
Bounce 2	…	…	…	…
Bounce …	…	…	…	…

The program should stop when the last height h2 is less than 1% of the initial height h_1.

Your report should include the program appropriately documented, the output and comments on the correctness of the program.

P1.2 (30 marks)

The CPU of calculators/computers has built-in hardware for simple arithmetic (e.g. +, -, * and /). More complex operations are effected by software based on certain algorithms which use simple arithmetic. For example, to compute $f(x) = \ln(x_o + 1)$ where $x_o = x - 1$ with $x > 0$, the following algorithm may be used:

$x_1 \leftarrow \dfrac{2x_o}{1+\sqrt{1+2^{-0}x_o}}$ $x_2 \leftarrow \dfrac{2x_1}{1+\sqrt{1+2^{-1}x_1}}$	In general: $x_{n+1} \leftarrow \dfrac{2x_n}{1+\sqrt{1+2^{-n}x_n}} \qquad (n = 0,1,2,\cdots)$

With increasing n, the above "*recurrence relation*" or "*iterative formula*" converges to $f(x)$; i.e. as n becomes larger, x_{n+1} approaches $\ln(x_o+1)$

Write the program which uses the above algorithm:

i) To compute $f(x)=\ln(x_o+1)$ for $0 \le x_o \le 10$ in step of 0.5 while ensuring Error[1] less than 1e-6.
ii) and print the results in tabular form with proper headings as follows:

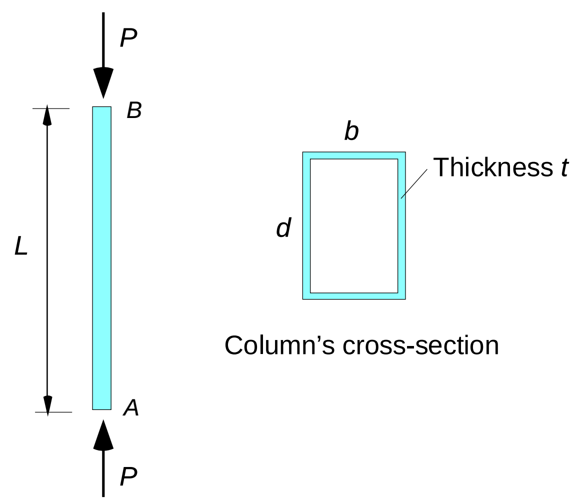
xo	x_{n+1}	Est. Error	Actual Error	$n+1$
0	0	0	0	1
0.5	…	…	…	…
…	…	…	…	…
10	…	…	…	…

where Actual Error is $|\ln(x_o+1)-x_{n+1}|$ and $n+1$ is the number of iterations used.

Your report should include the program appropriately documented, output and comments on the correctness of the program.

P1.3 (40 marks)

The steel column AB of length L = 3.8 m supports the force P = *1000 kN* as shown. The column's cross-section is a rectangular tube made up of steel plates of thickness t = 20mm.



Column supporting load P

The object of the design is to find the lightest tube (i.e. minimum $b + d$) that meets the following design conditions (i.e. constraints):

[1] The error in x_{n+1} may be estimated as $|x_{n+1}-x_n|$

- Depth/Width: $1 \le \frac{d}{b} \le 1.25$
- $P \le s A$

where A is the cross-sectional area $A = bd - (b-2t)(d-2t)$. The value of s is[*] calculated as follows for any given tube dimensions b, d.

$$I_x = \frac{bd^3}{12} - \frac{(b-2t)(d-2t)^3}{12}$$
$$I_y = \frac{db^3}{12} - \frac{(d-2t)(b-2t)^3}{12}$$

$$r_x = \sqrt{\frac{I_x}{A}} \quad , \quad r_y = \sqrt{\frac{I_y}{A}}$$

$E = 200\,\text{GPa}$, s_y = 250 MPa, F = 2

$$k_c = \sqrt{\frac{E\pi^2}{s_y / F}}$$

$$k = \text{the larger of } \begin{cases} \dfrac{1.2L}{r_x} \\ \dfrac{L}{r_y} \end{cases}$$

$$\text{If } k \le k_c : \begin{cases} G = \dfrac{5}{3} + \dfrac{3k}{8k_c} - \dfrac{1}{8}\left(\dfrac{k}{k_c}\right)^3 \\ s = \dfrac{s_y}{G}\left[1 - \dfrac{1}{F}\left(\dfrac{k}{k_c}\right)^2\right] \end{cases}$$

$$\text{If } k > k_c : \begin{cases} G = \dfrac{23}{12} \\ s = \dfrac{s_y}{FG}\left(\dfrac{k_c}{k}\right)^2 \end{cases}$$

First, write the program that will, for any given tube dimensions b and d, compute the above quantities. The program should detect and issue a message if any of the preceding constraints is violated.

Use your program to get the results for different trial tube dimensions, and select the "best design" by inspection.

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[*] s is the so-called “allowable stress” for the material of the column.

Once the program works well, insert two nested loops to generate suitable values of b and d in order to investigate the best plausible designs. The program should save the lightest feasible design.

Your report should include the final program appropriately documented, one page of the most relevant output that includes your best design, and comments on the correctness of your program as well as of your best design.