

**Advanced Microeconomic Theory**

**Examination I**

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**Duration: 1 hour 20 minutes**

1. Let  $(x_1, x_2, x_3)$  be the input vector and  $y$  be the output. Define the *input requirement set*, and explain what it means. Write down the input requirement set associated with the production technology  $y = x_1^a x_2^b x_3^c$ .

2. Consider the Leontief production technology represented by

$$q = f(x_1, x_2) = \min\{ax_1, bx_2\}.$$

- (a) Does this technology exhibit (global) constant returns to scale, increasing returns to scale, or decreasing returns to scale? Be sure to demonstrate your answer.  
(b) Set up the cost-minimization problem, and derive the cost function associated with this production technology. Verify that this cost function is homogeneous of degree 1 in  $(w_1, w_2)$ .

3. Suppose a firm's production function is given by

$$q = 3 \ln x.$$

Let  $p$  denote the price of output,  $w$  the price of input. Suppose  $p > w$ .

- (a) Derive the input demand function and output supply function.  
(b) Verify that the profit function associated with this production technology is convex in  $(p, w)$ .

4. A firm's production technology is represented by

$$q = f(x_1, x_2) = kx_1x_2(2 + x_1x_2)^{-1},$$

where  $k$  is a positive constant and  $q < k$ .

- (a) Calculate the elasticity of substitution between the two inputs.  
(b) Calculate the elasticity of scale.  
(c) Find the range of output ( $q$ ) over which this technology exhibits local increasing returns to scale.