ELG 3316 : *Electric Machines & Power Systems*

Mid-Term Examination Fall 2015

Date : 19 October 2015 Time : 16:00 - 17:15 (75 Minutes)

- Closed book examination. The only information sheets allowed are those included with the examination question paper.
- Calculators are allowed.
- There are **three questions** with unequal weights, **totaling 58 marks**. You should attempt **all** questions.
- It is strongly recommended that you write down your solutions step by step. This is the only way it is possible for you to earn marks step by step.
- **Units** must be given with all **final** numerical answers. Marks will be subtracted if units are omitted.
- Some questions require explanations rather than numerical calculations. Use the number of assigned marks as an indication of what is required. If such a question is worth 3 marks, then the answer must contain three independent pieces of information to earn all 3 marks.
- Read the questions carefully. This will prevent you from providing answers to questions that were not asked.
- Cheating will be penalised in accordance with University of Ottawa regulations.

Question 1 (19)

(a). Give two reasons why electrical power is transmitted/distributed using a three-phase configuration. **[2]**

(b). Write the phasor forms ($\tilde{V}$ and $\tilde{I}$, respectively) for the voltage $v(t) = 181\cos(377t - 33^\circ)$ and current $i(t) = \sqrt{2}\ 12.7\ \sin(377t + 5^\circ)$. If these are the voltage across, and current into, a single-phase load, what is the power factor angle of the load, and what is the reactive power consumed by the load? **[6]**

(c). What physical quantity does the number 377 represent in the expressions in (c), and what are its units? Be precise in your answer. **[1]**

Next consider the physical magnetic circuit structure shown in Fig.1. The core has a rectangular cross-section. The depth into the page is 2 cm. Flux leakage and fringing can be ignored. The permeability of the magnetic core material can be considered to be infinite (∞).

(d). Draw the magnetic equivalent circuit for the structure in Fig.~~2~~1. Fill in symbols for all circuit components and field quantities that you will use in answering the questions (e) and (f) below. **[2]**

(e). Find the flux-linkage of the winding for a DC current $I_{DC} = 5A$. (N = 100) **[5]**

(f). What is the flux-density in the 2mm air gap? **[3]**

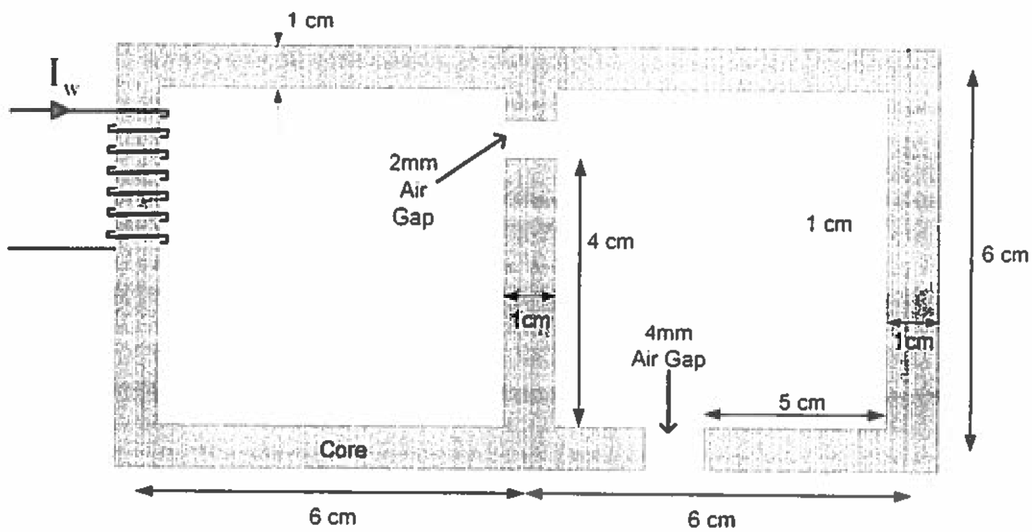


Figure 1 : Physical magnetic circuit structure for Question 1. The size of the air gaps is shown exaggerated.

Question 2 (18)

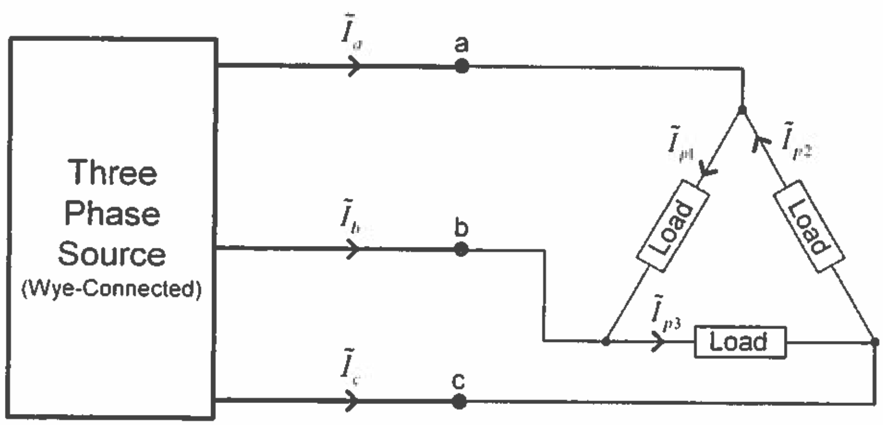


Figure 2 : Balanced three-phase source connected to a balanced three-phase load

Fig.2 shows a balanced 69 kV (line-to-line) three-phase source connected directly to the balanced three-phase load shown. The three-phase load consumes a total active power of 240 MW at a leading power factor of 0.8. All symbols used in answering the questions below must be shown on an appropriate circuit diagram.

(a). What is the apparent power, and reactive power, consumed by the three-phase load? [4]

(b). Calculate the complex line currents $\tilde{I}_a$ and $\tilde{I}_c$. [7]

(c). Determine the complex voltage $\tilde{V}_{ab}$. [2]

(d). Determine the complex current $\tilde{I}_{p3}$. [5]

Question 3 (21)

A 60 Hz single–phase source of voltage $\tilde{V}_1$ is connected to the primary terminals of a single-phase transformer. The secondary terminals of the transformer are connected to a purely resistive load that consumes 30 kW of power. The magnitude of the current through the load is measured to be 50A. The approximate equivalent circuit of the transformer consists of:

- A core loss resistance of 159 kΩ
- A magnetizing reactance of 40kΩ
- An equivalent winding loss resistance (referred to the primary) of 38Ω.
- An equivalent leakage reactance (referred to the primary) of 192Ω.
- An ideal transformer of turns-ratio 10:1

(a). Draw a diagram of the approximate equivalent circuit of the transformer, along with the source and load. The numerical values of all components must be shown on the circuit diagram, along with any current, voltage and component symbols you use in answering the questions that follow. [3]

(b). What is the power factor of the load? [1]

(c). What is the magnitude of the voltage across the load? [1]

(d). Find the complex supply voltage $\tilde{V}_1$ connected to the primary terminals of the transformer. [7]

(e). Determine the total power loss ("copper loss") in the transformer windings. [2]

(f). Calculate the power loss in the transformer core material. [3]

(g). What is the value of the overall leakage inductance of the transformer? [1]

(h). Calculate the complex the magnetising current of the transformer? [3]

Some General Information

$\mu_0 = 4\pi \times 10^{-7}$ H/m $\quad \lambda = N\Phi \quad L = \lambda / i \quad \Re = \ell / \mu A \quad e = d\lambda / dt \quad \sin x = \cos(x - 90^\circ)$

$I_{Line} = \sqrt{3}\, I_{Phase}$ (Delta) $\quad V_{Line-to-Line} = \sqrt{3}\, V_{Phase}$ (Wye) $\quad Z_Y = Z_\Delta / 3 \quad \phi_{pf} = \phi_v - \phi_i$

$\tilde{V}_{ab} = \sqrt{3}\, \tilde{V}_a e^{j30}$ (Wye Source) $\quad \tilde{I}_a = \sqrt{3}\, \tilde{I}_{ab} e^{-j30}$ (Delta Source) $\quad \tilde{S} = \tilde{V}\tilde{I}^* = P + jQ$

$$\frac{\tilde{E}_1}{\tilde{E}_2} = \frac{N_1}{N_2} = \frac{\tilde{I}_2}{\tilde{I}_2'} \qquad Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

QUESTION 1

19

2 (a). See Notes

6 (b).

• $v(t) = 181 \cos(377t - 33^\circ)$

$$\therefore \tilde{V} = \frac{181}{\sqrt{2}} e^{-j33^\circ} = 128\, e^{-j33^\circ} \text{ Volts}$$

(2)

$107.35 - j69.71$

$128 \angle -33^\circ$

• $i(t) = \sqrt{2}(12.7) \sin(377t + 5^\circ)$

$= \sqrt{2}(12.7) \cos(377t + 5^\circ - 90^\circ)$

$= \sqrt{2}(12.7) \cos(377t - 85^\circ)$

$$\therefore \tilde{I} = \frac{\sqrt{2}(12.7)}{\sqrt{2}} e^{-j85^\circ} = 12.7\, e^{-j85^\circ} \text{ Amps.}$$

(2)

$1.10 - j\,12.65$

$12.7 \angle -85^\circ$

• $\phi_{pf} = \phi_v - \phi_i = -33^\circ - (-85^\circ) = 52^\circ$ (1)

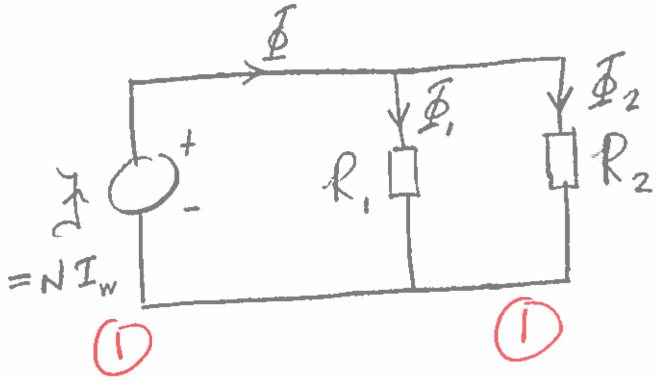
• $Q = |\tilde{V}||\tilde{I}| \sin\phi_{pf} = (128)(12.7) \sin 52^\circ = 1280.99$ VARs (1)

1 (c).

$\omega = 2\pi f = 377$ rad/sec (1)

ω ← Angular frequency; f ← 60 HZ

2 (d).



$A_g = 2\times10^{-2}\times10^{-2} = 2\times10^{-4}\ m^2$

$l_1 = 2mm = 2\times10^{-3}\ m$

$l_2 = 4mm = 4\times10^{-3}\ m$

$N = 100$

5 (e).

$\mathcal{F} = N I_w = (100)(5) = 500$ (1/2)

$R_1 = \dfrac{l_1}{\mu_0 A_g} = \dfrac{2\times10^{-3}}{4\pi\times10^{-7}\times2\times10^{-4}} = 79.5775\times10^{5}$ (1)

$R_2 = \dfrac{l_2}{\mu_0 A_g} = 2R_1$ (1)

"In Parallel With"

$R_{TOT} = R_1 \| R_2 \Rightarrow \dfrac{1}{R_{TOT}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{3}{2}\dfrac{1}{R_1}$

$\therefore R_{TOT} = \dfrac{2}{3}R_1 = \left(\dfrac{2}{3}\right)(79.5775\times10^{5}) = 53\times10^{5}$ (1/2)

$\therefore \Phi = \dfrac{\mathcal{F}}{R_{TOT}} = \dfrac{500}{53\times10^{5}} = 9.43\times10^{-5}\ Wb.$ (1)

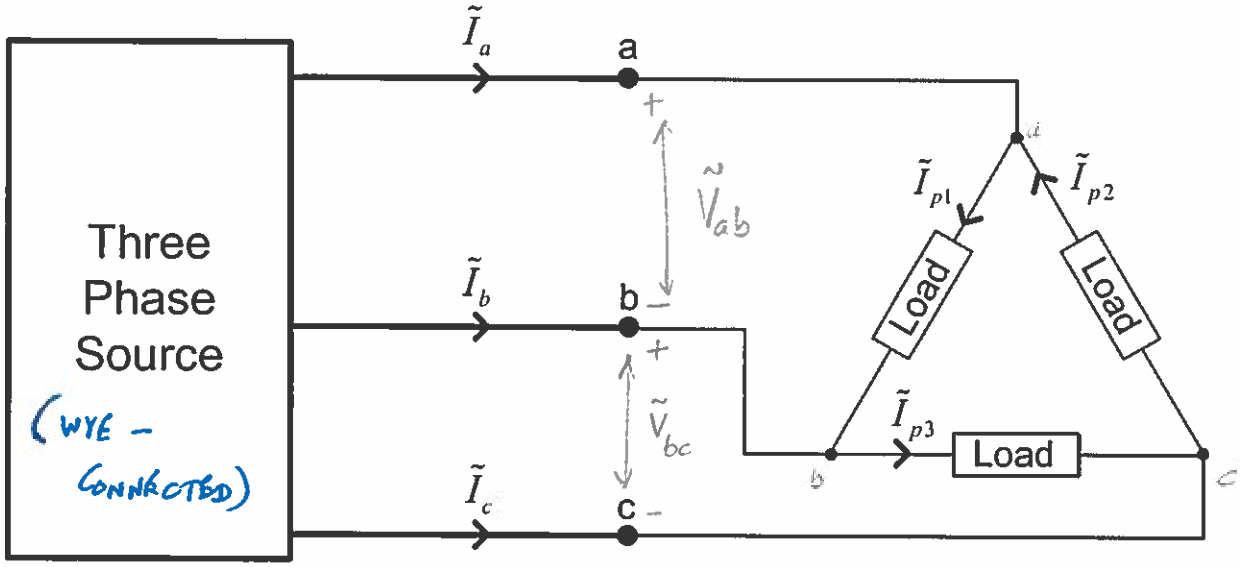
$\therefore \lambda = N\Phi = 9.43\times10^{-3}\ Wb$ (1)

3 (f).

$\Phi_1 = \dfrac{\mathcal{F}}{R_1} = \dfrac{500}{79.5775\times10^{5}} = 6.2832\times10^{-5}$ (1)

$\therefore B_1 = \Phi_1 / A_g = \dfrac{6.2832\times10^{-5}}{2\times10^{-4}} = 0.3142\ Tesla.$ (1) (1)

QUESTION 2   18



Given: $|\tilde{V}_{ab}| = 69 \text{ kV}$

$P = 240 \text{ MW}$

$pf = 0.8$ (Leading)

4 (a)

- $\cos\phi_{pf} = 0.8 \Rightarrow \phi_{pf} = \pm \cos^{-1} 0.8 = \pm 36.87°$ (1/2)

We are told that it is a leading pf, so must have $\phi_{pf} = -36.87°$ (1/2)

- $|\tilde{S}| = S = \frac{P}{pf} = \frac{240}{0.8} = 300 \text{ MVA}$ (1)

- $Q = S \sin\phi_{pf} = 300 \sin(-36.87°) = -180 \text{ MVAR}$ (1) (1)

7 (b). $|\tilde{V}_a| = \frac{69}{\sqrt{3}} \text{ kV} = 39.8372 \text{ kV}$ (1)

Assume reference phase on $\tilde{V}_a$, so that $\tilde{V}_a = 39.84\angle 0^\circ$ kV (1)

Now $3|\tilde{V}_a||\tilde{I}_a| = |\tilde{S}| \Rightarrow |\tilde{I}_a| = \frac{|\tilde{S}|/3}{|\tilde{V}_a|} = 2.5102$ kA. (1)

$\therefore \tilde{I}_a = 2.5102\angle 36.87^\circ$ kA (2)

$\therefore \tilde{I}_c = 2.5102\angle 36.87^\circ - 240^\circ = 2.5102\angle -203.12^\circ$ kA (1) (1)

(c). [2]

$\tilde{V}_{ab} = \sqrt{3}\,\tilde{V}_a e^{j30^\circ}$ for a wye-source (1)

$\therefore \tilde{V}_{ab} = \sqrt{3}(39.84)\angle 0^\circ + 30^\circ = 69\angle 30^\circ$ kV (1)

(d) [5]

$\tilde{I}_{p3} = ?$

We know that $|\tilde{V}_{bc}||\tilde{I}_{p3}|\cos\phi_{pf} = \frac{P}{3}$. (1)

($\cos\phi_{pf}$ = 0.8; P = 240 MW)

$\tilde{V}_{bc} = \tilde{V}_{ab}e^{-j120^\circ}$

$= 69\angle -90^\circ$ kV

$\therefore |\tilde{V}_{bc}| = 69$ kV

$\therefore |\tilde{I}_{p3}| = \frac{240/3}{(69)(0.8)} = 1.4493$ kA (1)

We know that $\tilde{I}_{p3}$ leads $\tilde{V}_{bc}$ by 36.87°. (1)

$\therefore \tilde{I}_{p3} = 1.4493\angle -90^\circ + 36.87^\circ$

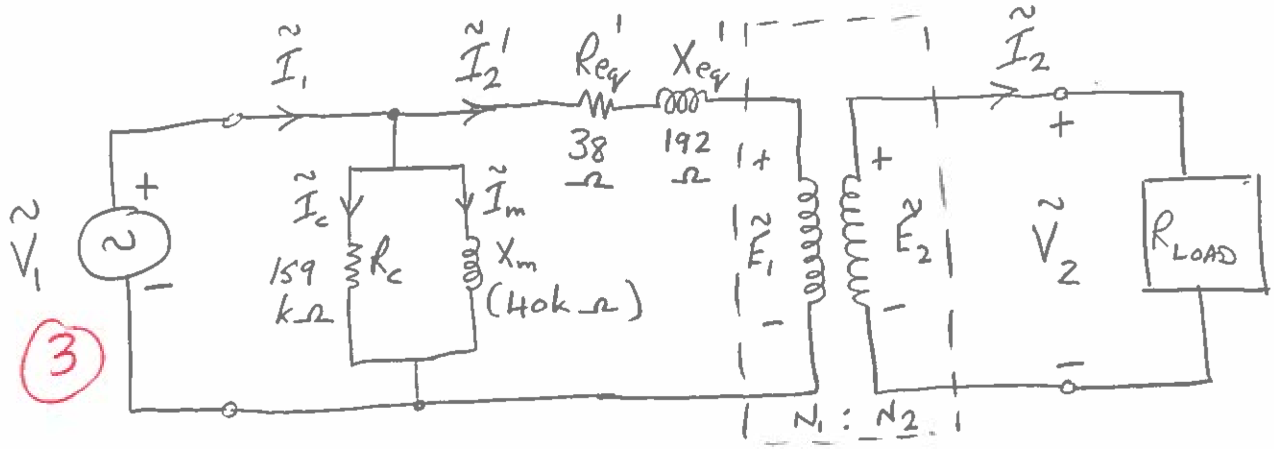
$= 1.4493\angle -53.13^\circ$ kA

(1) (1)

QUESTION 3

21

(a). 3



3

1 (b). Resistive load $\Rightarrow$ PF = 1.0 (1)

(c). 1

$P_{LOAD} = 30$ kW }
$|\tilde{I}_2| = 50$ A } Given

PF = 1.0 } From (b).

(1)

$$\underbrace{|\tilde{V}_2|}_{?}\ \underbrace{|\tilde{I}_2|}_{50}\ \underbrace{PF}_{1.0} = \underbrace{P_{LOAD}}_{30\times10^3} \Longrightarrow \underbrace{|\tilde{V}_2|}_{} = 600 \text{ Volts}$$

← Magnitude of Voltage Across Load.

(d). 7

$$|\tilde{I}_2'| = \underbrace{\left(\frac{N_2}{N_1}\right)}_{1/10}\underbrace{|\tilde{I}_2|}_{50} = 5$$ (1)

$$|\tilde{E}_1|\ \underbrace{|\tilde{I}_2'|}_{5}\ \underbrace{PF}_{1} = \underbrace{P_{LOAD}}_{3\times10^3} \Longrightarrow |\tilde{E}_1| = 6000$$ (1)

We assume $\tilde{I}_2'$ is the phase reference, so $\tilde{I}_2' = 5\angle 0°$ (1)

The "source" $(\tilde{E}_1, \tilde{I}_2')$ sees a PF $= 1.0$, so (1)

$$\tilde{E}_1 = |\tilde{E}_1| \angle 0° = 6000\angle 0°$$ (1)

KVL gives $\tilde{V}_1 - \tilde{I}_2'(R_{eq}' + jX_{eq}') - \tilde{E}_1 = 0$

$$\therefore \tilde{V}_1 = \tilde{I}_2'(R_{eq}' + jX_{eq}') + \tilde{E}_1$$ (1)

$$= 5(38 + j\,192) + 6000$$

$$\therefore \tilde{V}_1 = \underbrace{6190 + j\,960}_{6264\angle 8.8°} \text{ Volts}$$ (1)

This is the supply voltage.

(e). [2]

Power loss in transformer windings $= |\tilde{I}_2'|^2 R_{eq}'$ (1)

$$= (5)^2(38)$$

$$= 950 \text{ Watts}$$ (1)

(f). [3]

Power loss in the core $P_{CORE} = |\tilde{I}_c|^2 R_c$. (1/2)

Now $\tilde{I}_c = \tilde{V}_1 / R_c \Rightarrow |\tilde{I}_c| = |\tilde{V}_1| / R_c$ (1/2)

$$\therefore |\tilde{I}_c| = 6264 / 159\times10^3 = 0.0394$$ (1)

$$\therefore P_{CORE} = (0.0394)^2(159\times10^3) = 246.82 \text{ Watts.}$$ (1)

1 (g). $X'_{eq} = \omega L'_{eq} \Rightarrow L'_{eq} = X'_{eq}/\omega$ ← $\omega = 2\pi f$

(1)

$$= \frac{192}{2\pi(60)}$$

$$= 0.51 \ H.$$

3 (h). $\tilde{I}_m$ (See circuit diagram) is the magnetising current. (1)

$$\tilde{I}_m = \frac{\tilde{V}_1}{jX_m} = \frac{6190+j960}{j\,40\times10^3} = \underbrace{0.024 - j\,0.1548}_{0.1566 \angle -81.18^\circ}$$

(1) (1)