ENGR-233: Applied Advanced Calculus Winter 2014

Midterm test solutions

Variant A.

Problem 1. Find the parametric equation of the line of intersection of the two planes:

$P_1: x+y-8z=4$ and $P_2: 3x-y+4z=0$.

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Problem 2. Position vector of a moving particle is given by $\boldsymbol{r}(t)=\left(3t^2+1, 2t^2-7t+3, (t-1)^2\right)$.

(a) At what time(s) does the particle pass the xz -plane?

(b) What are the particle (i) coordinates, (ii) velocity, (iii) speed, (iv) acceleration at $t=2$?

Problem 3. Find the directional derivative of $F(x, y, z)=15x^2e^{-z}+3y^2$ in the direction $\boldsymbol{u}=(4,-4,2)$ at the point $(1,2,0)$.

Problem 4. Let $\boldsymbol{F}=\left(x(x^2+y^2+z^2)^m, y(x^2+y^2+z^2)^m, z(x^2+y^2+z^2)^m\right)$.
(a) Find $\nabla\cdot\boldsymbol{F}$; (b) Find m such that $\nabla\cdot\boldsymbol{F}=0$ for $x^2+y^2+x^2>0$.

Problem 5. Let

$$\boldsymbol{F}(x, y, z) = (a\cos y + b\sin z,\ c\cos z + d\sin x,\ e\cos x + f\sin y) .$$

(a) Find $\nabla \times \boldsymbol{F}$; (b) Find the values of a, b, c, d, e, f such that $\nabla \times \boldsymbol{F} \equiv \boldsymbol{F}$.

Problem 6. Find the work done by the force $\boldsymbol{F}(x, y, z) = (x - y, x^2, -z)$ moving a particle along **a line segment** from a point $P(1,2,3)$ to a point $Q(2,1,2)$.

Hint: Find the parametric equation of the line connecting P and Q , then evaluate the integral.

Problem 7. Let $\boldsymbol{F}(x, y, z)=\left(y e^{xy}, x e^{xy}-\sin(y+z), 3z^2-\sin(y+z)\right)$.
(a) Show that $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$ is independent of the path;
(b) Compute the integral for any path C from the point $A(2,-1,1)$ to the point $B(3,2,-2)$.

Ask Merlin Tomorrow if solved ok.
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