UNIVERSITY OF OTTAWA

Department of Economics	**ECO 3139**	**Winter 2017**
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1. A homogeneous good industry is composed of 3 firms. You are given the following information on output, price and marginal cost of each firm:

$$q_1 = 20$$
$$q_2 = 40$$
$$q_3 = 120$$
$$p = 20.50$$
$$c_1 = 20$$
$$c_2 = 19.5$$
$$c_3 = 17.5$$

Remember that for each firm

$$\frac{p - c_i}{p} = \frac{\alpha_i}{\eta},$$

where α_i is the market share of firm i and η is the price elasticity of demand.

a) Calculate the 2-firm concentration ratio
b) Calculate the Herfindahl index.
c) Calculate the number equivalent. What does it mean?
d) Calculate the Lerner index of ach firm.
e) Calculate the price elasticity of demand.

<u>Solution</u>

a) $q_3 > q_2 > q_1$

Industry output $Q = q_3 + q_2 + q_1 = 120 + 40 + 20 = 180$

$$\alpha_3 = \frac{120}{180} = 0.6667;\ \alpha_2 = \frac{40}{180} = 0.2222;\ \alpha_1 = \frac{20}{180} = 0.1111$$

2 - firm concentration ratio = $\alpha_3 + \alpha_2 = 0.6667 + 0.2222 = 0.8889$

b)

$$HHI = \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = (0.6667)^2 + (0.2222)^2 + (0.1111)^2 = 0.4445 + 0.0494 + 0.0123 = 0.5062$$

c) $N = \frac{1}{I_{HH}} = \frac{1}{0.5062} = 1.98 \cong 2$

d) $L_1 = \frac{p - c_1}{p} = \frac{20.50 - 20}{20.50} = \frac{0.50}{20.50} \cong 0.0244$

$L_2 = \frac{p - c_2}{p} = \frac{20.50 - 19.5}{20.50} = \frac{1}{20.50} = 0.0487$

$L_3 = \frac{p - c_3}{p} = \frac{20.50 - 17.5}{20.50} = \frac{3}{20.50} = 0.1463$

e) $L_i = \frac{p - c_i}{p} = \frac{\alpha_i}{\eta} \Rightarrow \eta = \frac{\alpha_i}{L_i}$

Hence,

$\eta_1 = \frac{\alpha_1}{L_1} = \frac{0.1111}{0.0244} = 4.5533 \approx (4.55)$

$\eta_2 = \frac{\alpha_2}{L_2} = \frac{0.2222}{0.0487} = 4.5626 \approx (4.56)$

$\eta_3 = \frac{\alpha_3}{L_3} = \frac{0.6667}{0.1463} = 4.5571 \approx (4.56)$

2. a) The bicycle industry consists of seven firms. Firms 1, 2, 3, 4 each has 10% market share, and firms 5,6,7 each has 20% market share. Using the concentration measures, answer the following questions:

(i) Calculate 4-firm concentration ratio for this industry.
(ii) Calculate the Herfindahl index (I_{HH}) for this industry.
(iii) Now, suppose that firms 1 and 2 merge, so that the new firm will have a market share of 20%.

1) Calculate the post merger I_{HH}
2) Calculate the change in the I_{HH} caused by the merger.
3) Will this merger be challenged in the U.S.? In Canada?

Solution

2(a)

(i) $CR_4 = 3(20\%) + 10\% = 705 = 0.7$

(ii) $HHI = 4(0.1)^2 + 3(0.2)^2 = 4(0.01) + 3(0.04) = 0.04 + 0.12 = 0.16$

(iii)

(1) $HHI = (0.2)^2 + 2(0.1)^2 + 3(0.2)^2 = 0.04 + 0.02 + 0.12 = 0.18$

(2) $\Delta HHI = 0.18 - 0.16 = 0.02$

(3) Since and $\Delta HHI = 0.02 > 0 \Rightarrow$ this merger may be challenged.

3. a) A firm produces two goods using the cost function

$$C(q_1,q_2) = q_1^{1/4} + q_2^{1/4} - (q_1q_2)^{1/4}$$

(i) Assume $q_2 = 0$ (only for part i). Does this cost function exhibit economies of scale?

(ii) Does this function exhibit economies of scope?

(iii) Calculate the incremental cost of q_2

b) A firm produces cars, q_C, and trucks, q_T. Its costs are given by

$$C(q_C, q_T) = 50q_C + 60q_T - \frac{3q_Cq_T}{2}$$

Calculate the measure of economies of scope for 20 cars and 10 trucks. What does this answer tell you about the production of cars and trucks?

Solution

(a) $C(q_1,q_2) = q_1^{1/4} + q_2^{1/4} - (q_1q_2)^{1/4}$

(i) When $q_2 = 0$, $C(q_1,0) = q_1^{1/4}$

$$AC(q_1,0) = \frac{C(q_1,0)}{q_1} = \frac{q_1^{1/4}}{q_1} = q_1^{1/4}q_1^{-1} = q_1^{-\frac{3}{4}}$$

$$\frac{d(AC)}{dq_1} = \left(-\frac{3}{4}\right)q_1^{-\frac{3}{4}-1} = \left(-\frac{3}{4}\right)q_1^{-\frac{7}{4}} < 0$$

Hence, the cost function exhibits economies of scale in the production of q_1.

(ii) From the cost function, $C(q_1,0) = q_1^{1/4}$, $C(0,q_2) = q_2^{1/4}$

So, $C(q_1,0)+C(0,q_2)=q_1^{1/4}+q_2^{1/4}>C(q_1,q_2)\Rightarrow$ the cost function has economies of scope at every level of output.

(iii) Incremental cost of

$q_2 = IC_2 = C(q_1,q_2)-C(q_1,0)=q_1^{1/4}+q_2^{1/4}-(q_1q_2)^{1/4}-q_1^{1/4}=q_2^{1/4}-(q_1q_2)^{1/4}$

Solution 3 (b)

The measure of economies of scope is:

$$S_C=\frac{C(q_S,0)+C(0,q_T)-C(q_S,q_T)}{C(q_S,q_T)}$$

For $q_S=2$ and $q_T=25$,

$$C(2,0)=4+(2)^2=4+4=8$$
$$C(0,25)=4+\sqrt{25}=4+5=9$$
$$C(2,25)=7+(2)^2+\sqrt{25}=7+4+5=16$$

$$S_C=\frac{8+9-16}{16}=\frac{1}{16}\approx 0.0435>0$$

Since S_C is positive, production of shampoo and toothpastes exhibits economies of scope. The magnitude of S_C indicates the degree of economies of scope.

4. Consider a market in which all firms have the following total cost function.

$$C(q) = 150 + q + 0.5q^2$$

Calculate the minimum efficient scale (MES). What does the minimum efficient scale mean in this example ?

Solution

(a) $$C(q)=150+q+\frac{1}{2}q^2 \qquad (4.1)$$

$$AC=\frac{C(q)}{q}=\frac{150+q+\frac{1}{2}q^2}{q}=\frac{150}{q}+1+\frac{1}{2}q$$

For minimum efficient scale (MES), find out the smallest output for which the AC is minimized.

$$\frac{\partial(AC)}{\delta q} = -\frac{150}{q^2} + 0 + \frac{1}{2} = 0 \Rightarrow q^2 = 300 \Rightarrow q = \sqrt{300} = 10\sqrt{3} \approx 17.32$$, which is the MES.

In order to achieve the lowest level average cost, the smallest level of output that the firm must produce is 17.32.

5. A market in prefect competition has the inverse demand function p = 50 - y. The marginal cost of production is $10.

a) Calculate the equilibrium price and quantity. Calculate the monopoly power of the firm.
b) Calculate consumer surplus.

Now assume that firms merge to a monopoly. The merger improves the product, which changes the demand function to p = 75 - y. Costs remained unchanged.

c) Calculate the new equilibrium (price and quantity). Calculate the monopoly power of the firm.

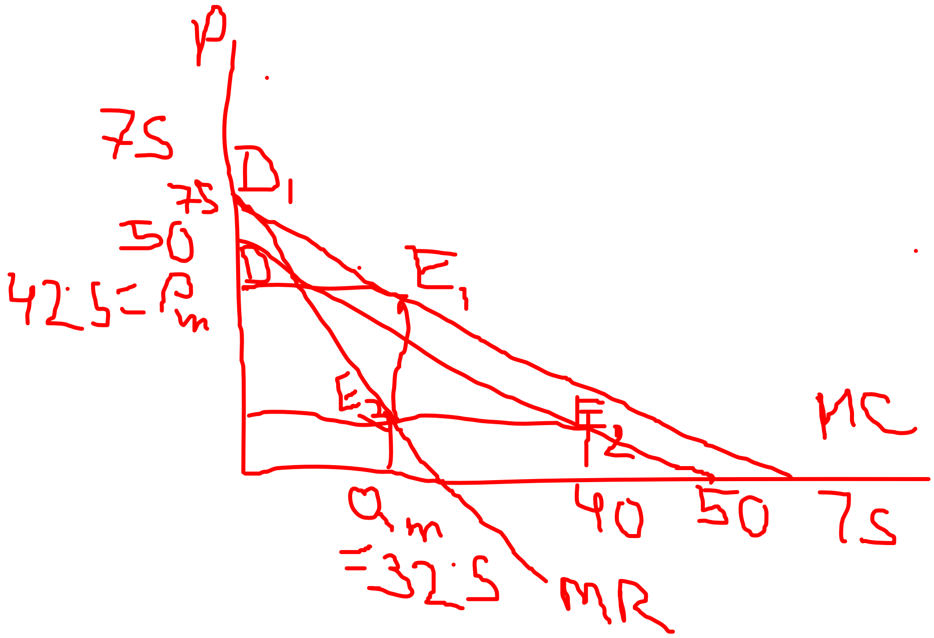
d) Calculate the new consumer surplus, profits, and total welfare.

e) Does total welfare increase or decrease with the merger?

<u>Solution</u>

Given $p = 50 - y$; $MC = AC = \$10$

(a) For perfectly competitive equilibrium, $p = MC \Rightarrow 50 - y = 10 \Rightarrow y = 50 - 10 = 40$ $p = 10$. Thus, $p = 10$, and $y = 40$



Monopoly power , i.e., Lerner index

$$L = \frac{P - MC}{P} = \frac{10 - 10}{10} = 0$$

(b) $CS^C = DE_2P_C = \frac{1}{2}(50 - 10)(40) = \frac{1}{2}(40)(40) = \frac{1}{2}(1600) = 800$

New demand is: $p = 75 - y$; $MC = AC = \$10$

Then $MR = 75 - 2y$

(c) Monopolist's profit maximizing condition is:

$MR = MC \Rightarrow 75 - 2y = 10 \Rightarrow 65 = 2y \Rightarrow y = \frac{65}{2} = 32.5$.

Then $p = 75 - 32.5 = 42.5$.

Thus, $p = 42.5$ and $y = 32.5$

Monopoly power , i.e., Lerner index

$$L = \frac{P - MC}{P} = \frac{32.5 - 10}{32.5} = \frac{22.5}{32.5} = 0.6923$$

(d) $CS^m = D_1E_1P_m = \frac{1}{2}(75 - 42.5)(32.5) = \frac{1}{2}(32.5)(32.5) = \frac{1}{2}(1056.25) = 528.125$

$$PS^m = \pi^m = (p^m - AC_m)y_m = (42.5 - 10)(32.5) = (32.5)(32.5) = 1056.25$$

Welfare = $W^m = CS^m + PS^m = 528.125 + 1056.25 = 1584.375$

(e) Total welfare under competition is: $W^C = CS^C + PS^C = 800 + 0 = 800$
Change in welfare = $\Delta W = W^m - W^C = 1584.375 - 800 = 784.375 > 0$
So, welfare increases.