GNG 1105 A

Mid-term Examination — Oct. 20, 2011

SOLUTIONS

1-

2 a) FBD - See Diagram

b) $R_x = -100 \times \frac{1}{1.414} = -70.7\,N$

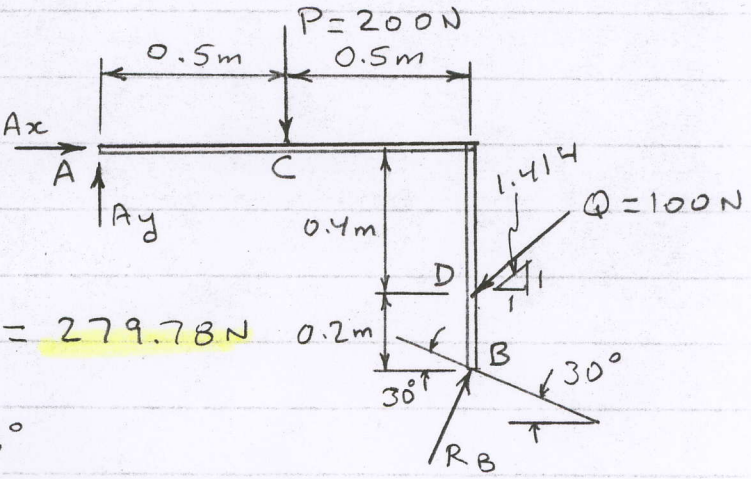
$R_y = -100 \times \frac{1}{1.414} - 200\,N$

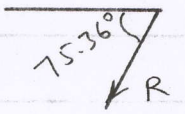
$= -270.7\,N$

$R = \sqrt{(-70.7)^2 + (-270.7)^2} = 279.78\,N$

$\theta = \tan^{-1}\frac{270.7}{70.7} = 75.36°$

$\therefore R = 279.78\,N$ ANS.





$+\circlearrowleft \Sigma M_A = -100 \times \frac{1}{1.414} \times 0.4m$

$-100 \times \frac{1}{1.414} \times 1.0m - 200 \times 0.5m = -198.98\,N.m.$ ANS.

$\therefore$ The force-couple system at A is: $R = 279.78\,N$ (75.36° ↘ R) ANS.

4 $M = 198.98\,N.m$ ↻

c) $+\circlearrowleft \Sigma M_A = -100 \times \frac{1}{1.414} \times 0.4m - 100 \times \frac{1}{1.414} \times 1.0m - 200 \times 0.5m$

$+ R_B \sin 30° \times 0.6m + R_B \cos 30° \times 1.0m = 0$

$1.166\,R_B = 198.98$; $\therefore R_B = 170.65\,N$ ANS.

$\pm \rightarrow \Sigma F_x = 0$

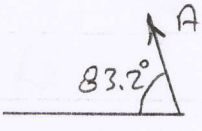
$A_x - 100 \times \frac{1}{1.414} + 170.65 \sin 30° = 0$, $\therefore A_x = -14.6\,N$

$+\uparrow \Sigma F_y = 0$

$A_y - 200N - 100 \times \frac{1}{1.414} + 170.65 \cos 30° = 0$; $\therefore A_y = 122.9\,N$

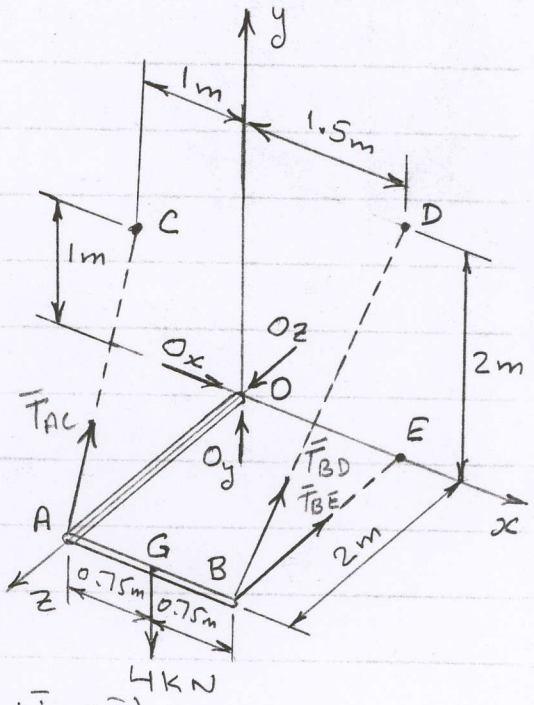
$A = \sqrt{(-14.6)^2 + (122.9)^2} = 123.76\,N$

4 $\theta = \tan^{-1}\frac{122.9}{-14.6} = 83.2°$



ANS.

3 2. a) FBD - See diagram



b) $\bar{AC} = -1m\,\bar{i} + 1m\,\bar{j} - 2m\,\bar{k}$

$AC = \sqrt{(-1)^2+(1)^2+(-2)^2} = 2.45\,m$

$\bar{BD} = +2m\,\bar{j} - 2m\,\bar{k}$

$BD = \sqrt{(2)^2+(-2)^2} = 2.83\,m$

$\bar{BE} = -2m\,\bar{k}$

$BE = 2.00\,m$

$\bar{T}_{AC} = T_{AC}\,\bar{\lambda}_{AC} = T_{AC}\,\frac{\bar{AC}}{AC} = \frac{T_{AC}}{2.45}(-1\bar{i}+1\bar{j}-2\bar{k})$

$\bar{T}_{BD} = T_{BD}\,\bar{\lambda}_{BD} = T_{BD}\,\frac{\bar{BD}}{BD} = \frac{T_{BD}}{2.83}(2\bar{j}-2\bar{k})$

6 $\bar{T}_{BE} = T_{BE}\,\bar{\lambda}_{BE} = T_{BE}\,\frac{\bar{BE}}{BE} = \frac{T_{BE}}{2.00}(-2\bar{k})$

c) $\Sigma \bar{M}_O = 0$

$\Sigma \bar{M}_O = \bar{r}_{A/O} \times \bar{T}_{AC} + \bar{r}_{B/O} \times \bar{T}_{BD} + \bar{r}_{B/O} \times T_{BE} + \bar{r}_{G/O} \times (-4KN)\bar{j} = 0$

where,

$\bar{r}_{A/O} = (2.0m)\bar{k}$; $\bar{r}_{B/O} = (1.5m)\bar{i} + (2.0m)\bar{k}$; $\bar{r}_{G/O} = (0.75m)\bar{i} + (2.0m$

$\therefore \Sigma \bar{M}_O = 2\bar{k} \times \frac{T_{AC}}{2.45}(-1\bar{i}+1\bar{j}-2\bar{k})$

$+ (1.5\bar{i}+2.0\bar{k}) \times \frac{T_{BD}}{2.83}(2\bar{j}-2\bar{k})$

$+ (1.5\bar{i}+2.0\bar{k}) \times \frac{T_{BE}}{2.00}(-2\bar{k})$

$+ (0.75\bar{i}+2.0\bar{k}) \times (-4\bar{j}) = 0$

$\Sigma M_O = -0.82\,T_{AC}\,\bar{j} - 0.82\,T_{AC}\,\bar{i}$

$+ 1.06\,T_{BD}\,\bar{k} + 1.06\,T_{BD}\,\bar{j} - 1.41\,T_{BD}\,\bar{i}$

$+ 1.5\,T_{BE}\,\bar{j}$

$- 3.0\bar{k} + 8.0\bar{i} = 0$

Equate coefficients of $\bar{i}$, $\bar{j}$ and $\bar{k}$ to zero

$\bar{i}$: $-0.82 T_{AC} - 1.41 T_{BD} + 8.0 = 0$ ——— (1)

$\bar{j}$: $-0.82 T_{AC} + 1.06 T_{BD} + 1.5 T_{BE} = 0$ ——— (2)

$\bar{k}$: $1.06 T_{BD} - 3.0 = 0$ ——— (3)

From Eq. (3) : $T_{BD} = \underline{2.83 \text{ KN}}$ ANS.

Insert value of T_{BD} in eq. (1) :

$-0.82 T_{AC} - 1.41 \times 2.83 + 8.0 = 0$

$0.82 T_{AC} = 4.01$

$\therefore T_{AC} = \underline{4.89 \text{ KN}}$ ANS.

Insert value of T_{AC} & T_{BD} in eq. (2) :

$-0.82 \times 4.89 + 1.06 \times 2.83 + 1.5 T_{BE} = 0$

$1.5 T_{BE} = 1.01$

$\therefore T_{BE} = \underline{0.67 \text{ KN}}$ ANS.

6

Determinant Form:

$$\Sigma M_O = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 1 & -2 \\ 0 & 0 & 2 \end{vmatrix} \times \frac{T_{AC}}{2.45} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 2 & -2 \\ 1.5 & 0 & 2 \end{vmatrix} \times \frac{T_{BD}}{2.83}$$

$$+ \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & -2 \\ 1.5 & 0 & 2 \end{vmatrix} \times \frac{T_{BE}}{2.00} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -4 & 0 \\ 0.75 & 0 & 2 \end{vmatrix} = 0$$