

Carleton University
 School of Mathematics and Statistics
 STAT 3502: Probability and Statistics - Assignment 1

Sections B due Tuesday, Jan. 31, 2017 In Tutorial

Section C due Monday, Jan. 30, 2017 In Tutorial

INSTRUCTIONS:

- I) Assignments are to be handed in prior to beginning of tutorial on the due dates listed above.
- II) Show all of your work. No credit will be given for answers without justification.
- III) No late assignments will be accepted.
- IV) Total of marks= 100

Last Name _____ **First Name** _____ **Student Number** _____

Question 1. [12 Marks] An airliner has 2 p.m., 8 p.m., and 10 a.m. flights from Montreal to Frankfurt, Paris, and Vancouver respectively. Let A denote the event “flight from Montreal to Frankfurt is full” and define B and C analogously for Paris and Vancouver. Assume $P(A) = 0.7$, $P(B) = 0.6$, and $P(C) = 0.8$ and that the three events are independent. Find the probability that

- [4] a. All flights are full.
- [4] b. At least one flight is full.
- [4] c. Only Vancouver flight is full.

Question 2. [8 Marks] Suppose that the proportions of blood types in a given population are as follows:

A	B	AB	O
0.4	0.12	0.05	0.43

Assuming that types of two randomly selected individuals are independent of one another, what is the probability that

- [4] a. both types are O?
- [4] b. both types match?

Question 3. [8 Marks] A utility company offers a lifeline rate to any household whose electricity usage falls below 240 kWh during a particular month. Let A denote the event that a randomly selected household in a certain community does not exceed the lifeline usage during January, and let B be the analogous event for the month of July (A and B refer to the same household). Suppose $P(A) = 0.8$, $P(B) = 0.7$, and $P(A \cup B) = 0.9$. Compute the following:

- [4] a. $P(A \cap B)$.

[4] b. The probability that the lifeline usage amount is exceeded in exactly one of the two months. Describe this event in terms of A and B.

Question 4. [12 Marks] A large company offers its employees two different health insurance plans and two different dental insurance plans. Plan 1 of each type is relatively inexpensive, but restricts the choice of providers, whereas plan 2 is more expensive but more flexible. The accompanying table gives the percentages of employees who have chosen the various plans:

Health Plan	Dental plan	
	1	2
1	27%	14%
2	24%	35%

Suppose that an employee is randomly selected and both the health plan and dental plan chosen by the selected employee are determined.

[4] a. What are the four possible outcomes (called also simple events)?

[4] b. What is the probability that the selected employee has chosen the more restrictive plan of each type?

[4] c. What is the probability that the employee has chosen the more flexible dental plan?

Question 5. [12 Marks] A small manufacturing company will start operating a night shift. There are 20 machinists employed by the company.

[4] a. If a night crew consists of 3 machinists, how many different crews are possible?

[4] b. If the machinists are ranked 1, 2, . . . , 20 in order of competence (1 being the best), how many of these crews would not have the best 10 machinists?

[4] c. How many of the crews would have at least 1 of the 10 best machinist?

Question 6. [12 Marks] At a certain gas station, 40% of the customers use regular unleaded gas, 35% use extra unleaded gas, and 25% use premium unleaded gas. Of those customers using regular gas, only 30% fill their tanks. Of those customers using extra gas, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.

[4] a. What is the probability that the next customer will request extra unleaded gas and fill the tank?

[4] b. What is the probability that the next customer fills the tank?

[4] c. If the next customer fills the tank, what is the probability that regular gas is requested? What is the probability that Extra gas is requested? What is the probability that Premium gas is requested?

Question 7. [4 Marks] Two pumps connected in parallel fail independently of one another on any given day. The probability that the older pump will fail is 0.15, and the probability that the newer pump will fail is 0.05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

Question 8. [4 Marks] An appliance dealer sells three different models of upright freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let X = the amount of storage space purchased by the next customer to buy a freezer. Suppose that X has pmf

x	13.5	15.9	19.1
$P(X = x)$	0.2	0.4	0.4

- [4] a. Compute the mean, variance, and standard deviation of X .
- [4] b. If the price of a freezer having capacity X cubic feet is $25X - 8.5$, what is the expected price paid by the next customer to buy a freezer?
- [4] c. What is the variance of the price paid by the next customer to buy a freezer?
- [4] d. Suppose that although the rated capacity of a freezer is X , the actual capacity is $h(X) = X - 0.01X^2$. What is the expected actual capacity of the freezer purchased by the next customer?

Question 9. [16 Marks] A mail-order computer business has five telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is as given in the accompanying table.

x	0	1	2	3	4	5
$P(X = x)$	0.1	0.15	0.2	0.25	0.22	0.08

Calculate the probability of each of the following events:

- [4] a. at most 3 lines are in use
- [4] b. fewer than 3 lines are in use
- [4] c. at least 3 lines are in use
- [4] d. between 2 and 5 lines, inclusive, are in use