

**Assignment Two Part Two:**

**Question One:**

**1)**

Probability model used is:

$$P(x) = 10Cx \cdot (0.2)^x \cdot (0.8)^{10-x}$$

Number of Premium Buses	Probability of outcome
x	P(x)
0	0.107
1	.268435456
2	.301989888
3	.201326592
4	.088080389
5	.026424115
6	.005505024
7	.000786432
8	.000073728
9	.000004096
10	.0000001024

**2)**

$$V(x) = (0 \cdot P(0)) + (1 \cdot P(1)) + (2 \cdot P(2)) + (3 \cdot P(3)) + (4 \cdot P(4)) + (5 \cdot P(5)) + (6 \cdot P(6)) + (7 \cdot P(7)) + (8 \cdot P(8)) + (9 \cdot P(9)) + (10 \cdot P(10)) = 2.0$$

Also can be performed by:

$$V(x) = n \cdot p \cdot (1-p) \\ = 2.0$$

2.0 is the expected value of premium buses.

**3)**

$$V(x) = n \cdot p \cdot (1-p) \\ = 10 \cdot 0.2 \cdot (1-0.2) \\ = 1.6$$

Therefore, the variance for the number of premium buses is 1.6 per month.

**4)**

The probability that the broker would send six or more premium buses for the month is:

Values taken from table in question one.

$$P(x \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10) = 0.0063693824 \\ 0.0063693824 \cdot 100 = 0.63693824\%$$

Therefore, the percentage change that the broker will send six or more premium buses is 0.63693824%.

**5)**

You would earn a maximum amount if you receive all premium buses because:

$$\$1500 \cdot 10 = \$15,000.$$

You would earn a minimum when you receive all economy buses because:

$$\$1000 \cdot 10 = \$10,000.$$

**6)**

Since the expected amount of premium buses you will receive is two that leaves eight of the buses to be economy buses, therefore:

$$((\# \text{ of premium buses}) \cdot 1500) + ((\# \text{ of economy buses}) \cdot 1000)$$

$$(2*1500)+(8*1000) = \$11000$$

\$11000 is the expected value for the monthly revenues from this broker.

**7)**

Variance of number of premium buses per month is 1.6

Variance of number of economy buses per month is 1.6

$$= n * p * (1-p)$$

$$= 10 * 0.8 * (1-0.8)$$

$$= 1.6$$

Monthly revenue is defined as  $1000y + 1500x$

$$\text{Variance of revenue is } 1000^2 * 1.6 + 1500^2 * 1.6 = \$5200000$$

$$\text{Standard deviation of monthly revenues from this broker is } \sqrt{5200000} = \$2280.35$$

Therefore, the standard deviation of the monthly revenues from this broker is \$2280.35

**8)**

For the scenario of 20 buses, expected value of the number of premium buses is  $0.2 * 20 = 4$ .

For the scenario of 30 buses, expected value of the number of economy buses is  $0.2 * 30 = 6$

This is because the probability of it being a premium bus has not changed.

**9)**

Variance of the number of premium buses:

$$= (\text{number of buses}) * (\text{probability of premium buses}) * (1 - (\text{probability of premium buses}))$$

For the scenario of 20 buses variance of the number of premium buses is  $20 * 0.2 * 0.8 = 3.2$

For the scenario of 30 buses variance of the number of premium buses is  $30 * 0.2 * 0.8 = 4.8$

**10)**

Some values taken from question eight.

For the scenario of 20 buses the expected value of number of premium buses per month is 4, therefore the expected value of number of economy buses per month is  $20 - 4 = 16$

$$\text{Expected value of revenue is } 1000 * 16 + 1500 * 4 = \$22000$$

For the scenario of 30 buses the expected value of number of premium buses per month is 6, therefore the expected value of number of economy buses per month is  $30 - 6 = 24$

$$\text{Expected value of revenue is } 1000 * 24 + 1500 * 6 = \$33000$$

**11)**

Some values taken from question nine.

For the scenario of 20 buses:

Variance of number of premium buses per month is 3.2.

Variance of number of economy buses per month is 3.2

Monthly revenue is  $1000y + 1500x$

$$\text{Variance of the revenue is } 1000^2 * 3.2 + 1500^2 * 3.2 = 10400000$$

$$\text{Standard deviation of monthly revenues from this broker for 20 buses is } \sqrt{10400000} = \$3224.90$$

For the scenario of 30 buses:

Variance of number of premium buses per month is 4.8

Variance of number of economy buses per month is 4.8

Monthly revenue is  $1000y + 1500x$

$$\text{Variance of revenue is } 1000^2 * 4.8 + 1500^2 * 4.8 = 15600000$$

$$\text{Standard deviation of monthly revenues from this broker for 30 buses is } \sqrt{15600000} = \$3949.68$$

**12)**

For the scenario of 20 buses:

Coefficient of variation is the standard deviation of revenue / expected value of revenue  
 $3224.90/22000 = 0.1466$   
Therefore, the coefficient of variation is 0.1466 for the scenario of 20 buses.

For the scenario of 30 buses:  
 $3949.68/33000 = 0.1197$   
Therefore, the coefficient of variation is 0.1197 for the scenario of 30 buses.

### **Question Two:**

#### **Plan:**

I need to determine whether I should continue operating as is, hire a fourth server, or rent a faster dispensing machine. To do this I will calculate the amount of time it takes the workers to serve customers and compare the results to one another. From there I will determine which is the better option between hiring the new employee or renting the faster dispenser in terms of response times to the customer. Then I will determine whether the cost is worth the amount both options cost or if the response time difference is negligible I will conclude that neither option should be chosen.

#### **Do:**

Using Poisson to determine the probability of each option being able to handle a growing amount of customers:

$$\text{Landa: } 1.7/1.3 = 1.308$$

#### **Base Option:**

$$\begin{aligned} ((1.3)^0 * e^{-1.3})/0! &= 0.2704 \\ ((1.3)^1 * e^{-1.3})/1! &= 0.3536 \\ ((1.3)^2 * e^{-1.3})/2! &= 0.2313 \\ ((1.3)^3 * e^{-1.3})/3! &= 0.1008 \end{aligned}$$

$$\begin{aligned} P(x \geq 3) &= 1 - P(x=0) + P(x=1) + P(x=2) + P(x=3) \\ &= 1 - 0.9561 \\ &= 0.0439 \\ 0.0439 * 100 &= 4.39\% \end{aligned}$$

Therefore, the base option (neither hiring another employee or investing in equipment) probability is 4.39%.

#### **Fourth Server Option:**

$$\begin{aligned} ((1.3)^0 * e^{-1.3})/0! &= 0.2704 \\ ((1.3)^1 * e^{-1.3})/1! &= 0.3536 \\ ((1.3)^2 * e^{-1.3})/2! &= 0.2313 \\ ((1.3)^3 * e^{-1.3})/3! &= 0.1008 \\ ((1.3)^4 * e^{-1.3})/4! &= 0.0324 \end{aligned}$$

$$\begin{aligned} P(x \geq 4) &= 1 - P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) \\ &= 1 - 0.998 \\ &= 0.01096 \\ 0.01096 * 100 &= 1.0906\% \end{aligned}$$

Therefore, the fourth employee's option probability is 1.0903%.

#### **Faster Dispensing Machine:**

$$\text{Landa} = 1.25/1.3 = 0.962$$

$$\begin{aligned} ((0.962)^0 * e^{-0.962})/0! &= 0.38212 \\ ((0.962)^1 * e^{-0.962})/1! &= 0.3676 \\ ((0.962)^2 * e^{-0.962})/2! &= 0.1768 \\ ((0.962)^3 * e^{-0.962})/3! &= 0.0567 \end{aligned}$$

$$\begin{aligned} P(x \geq 3) &= 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3) \\ &= 1 - (0.98322) \\ &= 0.01678 \\ 0.01678 * 100 &= 1.678\% \end{aligned}$$

Therefore, the faster dispenser machine's option probability is 1.678%.

**Report:**

All the calculations have determined that all of the options are acceptable because the probability of serving more than 3 customers all within 1.7 minutes of each other is less than 10%. However, the best option would be to hire a fourth employee at a cost of \$38,000 dollars annually because it has the lowest probability of 1.0906% of having more than three customers all within 1.7 minutes of each other thus determining that it can serve the most amount of people in the shortest amount of time thus making it more efficient. Whether the benefits outweigh the cost is subjective but since all meet the criteria of being below 10% I would suggest keeping the business the way it is because it is both the most cost efficient and below 10%.

**Personal Ethics Statement**

**Individual Assignment:**

By signing this Statement, I am attesting to the fact that I have reviewed the entirety of my attached work and that I have applied all the appropriate rules of quotation and referencing in use at the Telfer School of Management at the University of Ottawa, as well as adhered to the fraud policies outlined in the Academic Regulations in the University's Undergraduate Studies Calendar. [Academic Fraud Webpage](#)

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Signature

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