ENGR 213
Suggested for Review Problems

Problem 1.

Solve the following equations by the method of separation of variables:

(a) $y'(x) = 10^{x+y}$;

(b) $(1 + x^2)(1 + y^2)dx - xydy = 0$;

(c) $x^2 \frac{dy}{dx} = y - xy$;

(d) $(2y + 3)^2 dx - (4x + 5)^2 dy = 0$;

(e) $(2x + y)dx - xdy = 0$.

Problem 2.

Solve the following equations by the exact differentials:

(a) $(x + 2y - 2)dx + (2x - y + 3)dy = 0$;

(b) $(3x^2 + 4xy - y^2 - 2)dx + (2x^2 - 2xy + 3y^2 + 3)dy = 0$;

(c) $(1 + \ln x + \frac{y}{x})dx = (1 - \ln x)dy = 0$;

(d) $e^{-y}dx - (2y + xe^{-y})dy = 0$;

(e) $(3x^2y + xy^2)dx + (x^3 + x^2y)dy = 0$.

Problem 3.

Solve the following initial-value problem

(a) $y(x) = x(y'(x) - x\cos x)$, $y(\pi) = 1$;

(b) $\frac{dy}{dx} - 2y = e^x$, $y(0) = 3$;

(c) $y\frac{dx}{dy} - x = 2y^2$, $y(-1) = 2$.

Problem 4.

Solve the following Bernoulli equation:

(a) $\frac{dy}{dx} + 2y = e^x y^2$;

(b) $(2 + x^2)\frac{dy}{dx} + xy = x^3y^3$;

(c) $y'(x) + \frac{y}{x-2} = 5(x-2)y^{1/2}$;

(d) $y\frac{dx}{dy} + x - y^2x^2 = 0$.

Problem 5.

Solve by substitution

$$xy'(x) - y = (x+y)\ln\frac{x+y}{x}$$

Problem 6.

Solve the following equations by the method of integrating factor:

(a) $\frac{dy}{dx} - 2y = e^x$;

(b) $\frac{dy}{dx} - \frac{y}{x} = x^n$, $n \neq 0$;

(c) $xy'(x) + (3x+1)y = e^{-3x}$.

Problem 7.

Find the general solution of the following differential equations:

(a) $y''(x) - 4y'(x) + 3y(x) = 0$;

(b) $y''(x) + 3y'(x) + 2y(x) = 0$;

(c) $4y''(x) - 4y'(x) + y(x) = 0$.

Problem 8.

Solve the boundary-value problem:

(a) $y'' + 2y' + 10y = 0 \quad y(0) = 2, \quad y(\pi/2) = 3$.

Problem 9.

Solve the following differential equations by the method of undetermined coefficients:

(a) $y'' + 3y' + 4y = 3x + 2$;

(b) $y'' - 2y' + y = 2xe^x + e^x \sin 2x$;

(c) $y''(x) - y(x) = \cos x$;

(d) $y''(x) + y'(x) - 2y(x) = x^2$.

Problem 10.

Solve the following differential equations by variation of parameters:

(a) $y'' + 4y' + 4y = (1 + x)e^{3x}$;

(b) $y'' + y' - 6y = xe^{2x}$;

(c) $4y'' - 4y' + y = e^{x/2}\sqrt{1 - x^2}$;

(d) $y'' + y = \tan x$.

Problem 11.

Solve Cauchy-Euler equation:

(a) $x^2y'' + xy' - 4y = 10x$;

(b) $\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} - \frac{V}{r^2} = 0$.

Problem 12.

Given the equation of free damped motion

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 10x = 0, \quad y(0) = 3, \quad y'(0) = 0$$

Determine is it overdamped, underdamped, or critically damped motion. Find its amplitude, phase angle, period and frequency.

Problem 13.

The equation describing the motion of the mass-spring system is

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

where $k = 1N/m$ and $m = 1kg$. Find the position x of mass at an arbitrary time t if the initial position of the mass is $1m$ and the initial velocity is 0.

Problem 14.

Use the power series method to find the solution of the initial value problem. Write the first eight nonzero terms of the power series centered at $x = 0$.

(a) $y'' + 2xy' + y = 0, \quad y(0) = 1,\ y'(0) = 0$;

(b) $y'' - x^2y = 0, \quad y(0) = 1,\ y'(0) = 0$;

(c) $(x^2+1)y'' + 2xy' = 0, \qquad y(0) = 0,\ y'(0) = 1;$

(d) $y'' + x^2y' + 2xy = 0, \qquad y(0) = 1,\ y'(0) = 0;$

(e) $y'' - y = 0;$

(e) $y'' = e^y, \qquad y(0) = 0,\ y'(0) = -1.$

Problem 15.

Solve the following systems of differential equations:

(a) $\begin{aligned} \frac{dx}{dt} &= x + 2y \\ \frac{dy}{dt} &= -x + 3y \end{aligned}$

(b) $\begin{aligned} \frac{dx}{dt} &= x - y \\ \frac{dy}{dt} &= y - 4x + 3 \end{aligned}$

(c) $\begin{aligned} \frac{dx}{dt} &= x - 2y + 2 \\ \frac{dy}{dt} &= -2x + y \end{aligned}$

(d) $\begin{aligned} \frac{dx}{dt} &= x + y \\ \frac{dy}{dt} &= 3x - 2y \end{aligned}$

(e) $\begin{aligned} \frac{dx}{dt} &= y - 2\cos t \\ \frac{dy}{dt} &= 2x + y \end{aligned}$

(f) $\begin{aligned} \frac{dx}{dt} &= 2y - x \\ \frac{dy}{dt} &= 4y - 3x + \frac{e^{3t}}{e^{2t}+1} \end{aligned}$

(g) $\begin{aligned} \frac{dx}{dt} &= x - 2y + 2 \\ \frac{dy}{dt} &= -2x + y \end{aligned}$

Problem 16.

The molecular bond due to intermolecular forces is flexible. A diatomic molecule like Oxygen (O_2), if disturbed, will oscillate to and from the equilibrium position (R_0 or $x = 0$ minimum potential energy) approximated by the equation:

$$\mu \frac{d^2x}{dt^2} + kx = 0$$

where μ is the reduced mass of the system $\mu = m_{O2}/2$ and k is the spring constant. Explain why the motion of the two atoms will be oscillatory. If the reduced mass μ for the Oxygen molecule (O_2) is 1.33×10^{-26}kg and $k = 1195$ N/m. What is the natural frequency of O_2?