Tutorial 1: Process Modeling - Solutions

Q1. (Exercise 1.7 in textbook) *Two flow control loops are shown in the drawing. Indicate whether each system is either a feedback or a feedforward control system. Justify your answer. It can be assumed that the distance between the flow transmitter (FT) and the control valve is small in each system.*

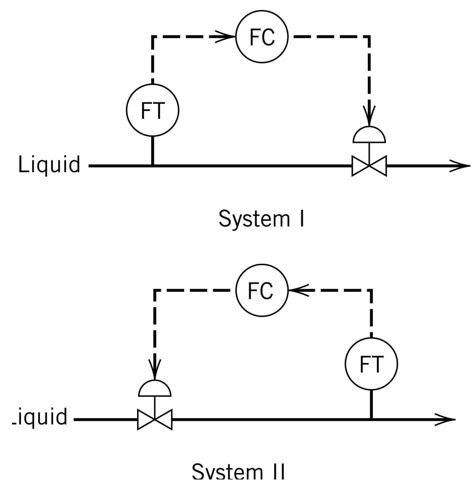

Figure 1: Question 1

Soultion: Both flow control loops are feedback control systems. In both cases, the controlled variable (flow) is measured and controller responds to that measurement. Here is a quick way to identify CVs, MVs and DVs: Ask yourself the following questions

- What variables am I interested in controlling? - The answer to this question provides CVs.
- What can I adjust to control CVs? - The answer to this question provides MVs.
- What other variables affect CVs and if you can adjust/manipulate them? - If you can not manipulate, call them DVs.

Now to identify whether a control system is feedback/feedforward ask yourself the following questions:

- Is my controller using CV measurements? - If yes, you have a FeedBack controller.
- Is my controller using DV measurements? - If yes, you have a FeedForward controller.

Q2. (Exercise 2.2 in textbook) *A completely enclosed stirred-tank heating process is used to heat an incoming stream whose flow rate varies. The heating rate from this coil and the volume are both constant.*

(a) *Develop a mathematical model (differential and algebraic equations) that describes the exit temperature if heat losses to the ambient occur and if the ambient temperature (T_a) and the incoming stream's temperature (T_i) both can vary.*

(b) *Discuss qualitatively what you expect to happen as T_i and w increase (or decrease). Justify by reference to your model.*

Notes: ρ and C_ρ are constants.
U, the overall heat transfer coefficient, is constant.

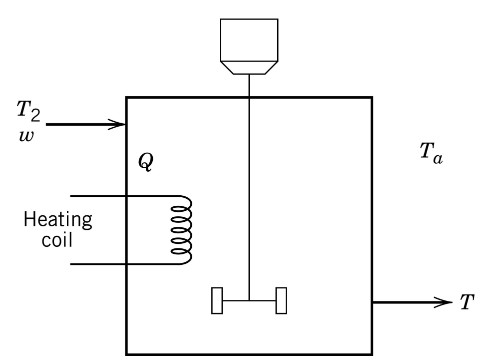

Figure 2: Question 2

A_s is the surface area for heat losses to ambient.
$T_i > T_a$ (inlet temperature is higher than ambient temperature).

Solution:

(a) Performing energy balance,

$$C_p \frac{d}{dt}[\rho V(T - T_{ref})] = wC_p(T_i - T_{ref}) - wC_p(T - T_{ref}) - UA_s(T - T_a) + Q$$

Simplifying,

$$\rho V C_p \frac{dT}{dt} = wC_pT_i - wC_pT - UA_s(T - T_a) + Q$$
$$\rho V C_p \frac{dT}{dt} = wC_p(T_i - T) - UA_s(T - T_a) + Q$$

(b) T increases if T_i increases and vice versa. T decreases if w increases and vice versa if $(T_i - T) < 0$. In other words, if $Q > UA_s(T - T_a)$, the contents are heated, and $T > T_i$.

Q3.(Exercise 2.13 in textbook) *The liquid storage tank shown below has two inlet streams with mass flow rates w_1 and w_2 and an exit stream with flow rate w_3. The cylindrical tank is 2.5m tall and 2m in diameter. The liquid has a density of 800 kg/m^3. Normal operating procedure is to fill the tank until the liquid level reaches a nominal value of 1.75m using constant flow rates: w_1 = 120 kg/min, $w_2 = 100$kg/min, and $w_3 = 200$kg/min. At that point, inlet flow rate w_1 is adjusted so that the level remains constant. However, on this particular day, corrosion of the tank has opened up a hole in the wall at a height of 1m, producing a leak whose volumetric flow rate q_4(m^3/min) can be approximated by (h is the height in meters).*

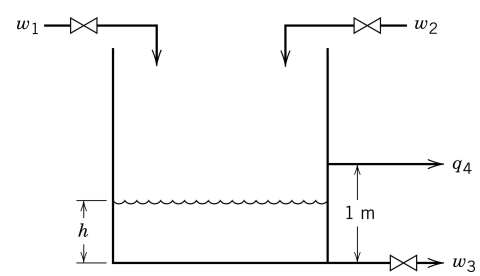

Figure 3: Question 3

$$q_4 = 0.025\sqrt{h - 1}$$

(a) *If the tank was initially empty, how long did it take for the liquid level to reach the corrosion point?*

(b) *If mass flow rates w_1, w_2, w_3 are kept constant indefinitely, will the tank eventually overflow? Justify your answer.*

Solution:

(a) Model of tank (normal operation):

$$\rho A \frac{dh}{dt} = w_1 + w_2 - w_3 \quad \text{(Below the leak point)}$$

$$A = \frac{\pi(2)^2}{4} = \pi = 3.14\ m^2$$

$$(800)(3.14)\frac{dh}{dt} = 120 + 100 - 200 = 20$$

$$\frac{dh}{dt} = \frac{20}{(800)(3.14)} = 0.007962\ m/min$$

Time to reach leak point ($h = 1\ m$) = 125.6 min.

(b) Model of tank with leak and w_1,w_2, w_3 constant:

$$\rho A\frac{dh}{dt} = 20 - \delta q_4 = 20 - \rho(0.025)\sqrt{h-1} = 20 - 20\sqrt{h-1}, \quad h \geq 1$$

To check for overflow, one can simply find the level h_m at which $\frac{dh}{dt} = 0$. That is the maximum value of level when no overflow occurs.

$$0 = 20 - 20\sqrt{h_m - 1} \quad \text{or} \quad h_m = 2\ m$$

Thus, overflow does not occur in presence of leak because $h_m < 2.25m$.

Q4. (Exercise 2.16 in textbook) *In medical applications the chief objectives for drug delivery are: (i) to deliver the drug to the correct location in the patient's body, and (ii) to obtain a specified drug concentration profile in the body through a controlled release of the drug over time. Drugs are often administered as pills. In order to derive a simple dynamic model of pill dissolution, assume that the rate of dissolution r_d of the pill in a patient is proportional to the product of the pill surface area and the concentration driving force:*

$$r_d = kA(c_s - C_{aq})$$

where C_{aq} is the concentration of the dissolved drug in the aqueous medium, c_s is he saturation value, A is the surface area of the pill, and k is the mass transfer coefficient. Because $c_s >> C_{aq}$, even if the pill dissolves completely, the rate of dissolution reduces to $r_d = kAc_s$.

(a) *Derive a dynamic model that can be used to calculate pill mass M as a function of time. You can make the following simplifying assumptions:*

   (i) *The rate of dissolution of the pill is given by $r_d = kAc_s$.*

   (ii) *The pill can be approximated as a cylinder with radius r and height h. It can be assumed that $h/r >> 1$. Thus the pill surface area can be approximated as $A = 2\pi rh$.*

(b) *For the conditions given below, how much time is required for the pill radius r to be reduced by 90% from its initial value of r_0?*
Notes:
ρ *= 1.2g/ml*
r_0 *= 0.4cm*
$h = 1.8cm$
c_s *= 500g/L*
k *= 0.016 cm/min.*

Solution:

(a) We can assume that ρ and h are approximately constant. The dynamic model is given by

$$r_d = -\frac{dM}{dt} = kAc_s \tag{1}$$

Notice that:

$$M = \rho V \qquad \therefore \frac{dM}{dt} = \rho \frac{dV}{dt} \tag{2}$$

$$V = \pi r^2 h \qquad \therefore \frac{dV}{dt} = (2\pi r h)\frac{dr}{dt} = A\frac{dr}{dt} \tag{3}$$

Substituting (3) into (2) and then into (1),

$$-\rho A \frac{dr}{dt} = kAc_s \qquad \therefore -\rho\frac{dr}{dt} = kc_s$$

Integrating,

$$\int_{r_0}^{r} dr = -\frac{kc_s}{\rho}\int_0^t dt \qquad \therefore r(t) = r_0 - \frac{kc_s}{\rho}t \tag{4}$$

Finally,

$$M = \rho V = \rho\pi h r^2$$

then

$$M(t) = \rho\pi h\left(r_0 - \frac{kc_s}{\rho}t\right)^2$$

(b) The time required for the pill radius r to be reduced to 90% is given by (4):

$$0.1r_0 = r_0 - \frac{kc_s}{\rho}t$$

$$\therefore t = \frac{0.9r_0\rho}{kc_s} = \frac{(0.9)(0.4)(1.2)}{(0.016)(0.5)} = 54\ min.$$