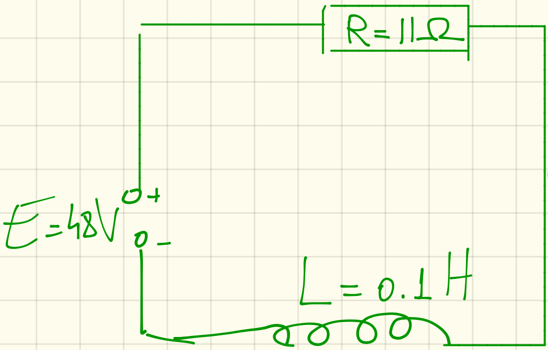


$$I(t) = ? \qquad I(0) = 0$$

$$(1)\quad E(t) = R\cdot I + L\frac{dI}{dt}$$

$$48 = 11 I + 0.1\frac{dI}{dt}$$

(1) → std. form (2) $\displaystyle \frac{dI}{dt} + \underbrace{\frac{R}{L}}_{P(t)} I(t) = \underbrace{\frac{E(t)}{L}}_{f(t)}$

$$h = \int P(t)\,dt \Rightarrow \quad e^{h} = e^{\int \frac{R}{L}dt} = e^{\frac{R}{L}t}$$

$$e^{-h} = e^{-\frac{R}{L}t}$$

$$I(t) = e^{-h}\left[\int f(t)e^{h}dt + c\right] \leftarrow \text{integration factor method}$$

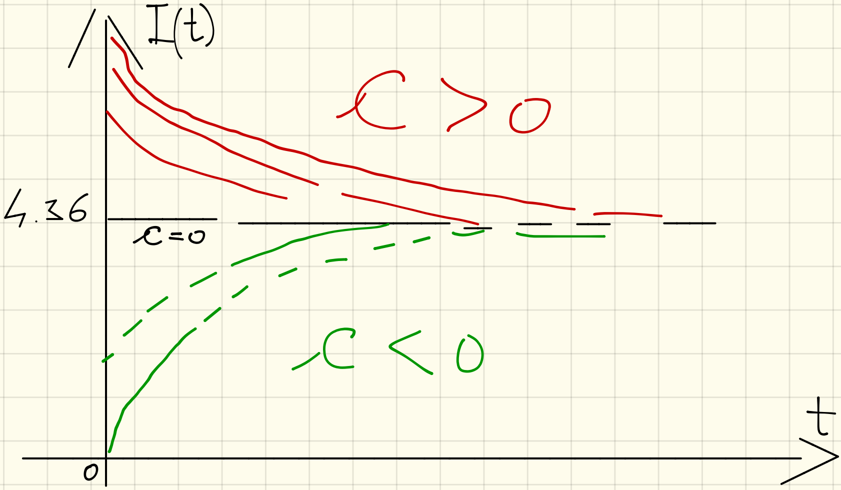
$$I = e^{-\frac{R}{L}t}\left[\underbrace{\int \frac{E}{L}e^{\frac{R}{L}t}dt}_{\frac{E}{L}\,\frac{1}{R/L}\,e^{\frac{R}{L}t}} + c\right] = e^{-\frac{R}{L}t}\left[\frac{E}{\cancel{L}}\cdot\frac{\cancel{L}}{R}e^{\frac{R}{L}t} + c\right]$$

$$I(t) = \frac{E}{R} + c\,e^{-\frac{R}{L}t}$$

$$I(t) = \frac{E}{R} + c\,e^{-\frac{R}{L}t}$$

$$I(0) = \frac{48}{11} + c\,e^{0} = 0 \Rightarrow c = -\frac{48}{11} \simeq -4.36$$

$$I(t) = 4.36\left(1 - e^{-110t}\right) \quad [A] \qquad t \in [0, \infty)$$



EXACT EQUATIONS

$y \cdot dx + x\,dy = 0 \Rightarrow d(x \cdot y) = 0 \Rightarrow x \cdot y = c$

$f(x,y) = z$ has continuous first PARTIAL derivatives; $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$

$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$ → total differential of $f(x,y)$

$f(x,y) = x^2 - 5xy + y^3$; $f(x,y) = c$ → arbitrary constant

$$d\big(f(x,y)\big) = \overbrace{(2x-5y)}^{\partial f/\partial x}dx + \overbrace{(-5x+3y^2)}^{\partial f/\partial y}dy = 0$$

DEF. of an EXACT DIFF

$M(x,y)dx + N(x,y)dy$ is an EXACT DIFFERENTIAL in a region $R \supset xy$-plane if it corresponds to the differential of some function $f(x,y)$

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad (3)$$

if $\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x} \Rightarrow$ (3) is an exact D.E.

Example:

$$\underbrace{2xy}_{M(x,y)}\,dx + \underbrace{(x^2-1)}_{N(x,y)}\,dy = 0$$

$$\frac{\partial M}{\partial y} = 2x$$
$$\frac{\partial N}{\partial x} = 2x$$
$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \text{this is an EXACT EQ.}$$

$\boxed{\text{find } f(x,y)} \rightarrow d(f(x,y)) = 0$; $\boxed{f(x,y) = c}$

$$\underbrace{\frac{\partial f}{\partial x} = M(x,y)}_{(4)} \; ; \; \underbrace{\frac{\partial f}{\partial y} = N(x,y)}_{(5)}$$

use (4): $\frac{\partial f}{\partial x} = 2xy \xrightarrow{\int \cdot} \int \frac{\partial f}{\partial x}\,dx = \int 2xy\,dx \Rightarrow f(x,y) = x^2y + h(y)$

use (5) $\cdot$ $\frac{\partial f}{\partial y} = x^2-1 \xrightarrow{\int \cdot} \int \frac{\partial f}{\partial y}\,dy = \int (x^2-1)\,dy \Rightarrow f(x,y) = (x^2-1)y + g(x)$

$$\frac{\partial f}{\partial y} = \frac{\partial(x^2y + h(y))}{\partial y} = x^2-1 \; ; \; x^2 + h'(y) = x^2 - 1 \Rightarrow h'(y) = -1$$

$$h(y) = -y + c_1 \qquad c = c_2 - c_1 \text{ notation}$$

$$f(x,y) = x^2y - y + c_1 = c_2 \Rightarrow \boxed{x^2y - y = c}$$

IMPLICIT SOL.

$$y = \boxed{y(x) = \frac{c}{x^2-1}}$$

EXPLICIT SOLUTION

$I = (-\infty, -1)$ OR $I = (-1, 1)$ OR $I = (1, \infty)$

Example: $\underbrace{xy}_{M(x,y)}\,dx + \underbrace{(2x^2+3y^2-20)}_{N(x,y)}\,dy = 0$ SOLVE w/ the exact diff. method

$$\frac{\partial M}{\partial y} \overset{?}{=} \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial(xy)}{\partial y} = x \quad ; \quad \frac{\partial N}{\partial x} = \frac{\partial(2x^2+3y^2-20)}{\partial x} = 4x$$

- if the given EQ does not check for EXACTNESS

Define integration factors:

$$\text{if } \frac{M_y - N_x}{N} = z(x) \rightarrow \text{use I.F.} = e^{\int z(x)\,dx}$$

or

$$\text{if } \frac{N_x - M_y}{M} = z(y) \rightarrow \text{use I.F.} = e^{\int z(y)\,dy}$$

$$\frac{\partial N(x,y)}{\partial x} = N_x \quad ; \quad \frac{\partial M(x,y)}{\partial y} = M_y$$

$$\frac{M_y - N_x}{N} = \frac{x - 4x}{2x^2+3y^2-20} \neq z(x) \quad ; \quad \text{but: } \frac{N_x - M_y}{M} = \frac{4x - x}{xy} = \underbrace{\frac{3}{y}}_{z(y)}$$

$$\text{I.F } e^{\int z(y)\,dy} = e^{\int \frac{3}{y}\,dy} = e^{3\ln y} = y^3$$

$$xy\,dx + (2x^2+3y^2-20)\,dy = 0 \;/\cdot\, y^3$$

$$xy^4\,dx + (2x^2y^3+3y^5-20y^3)\,dy = 0 \rightarrow \text{CHECK FOR EXACTNESS of the new D.E.}$$