ENGR 213 - APPLIED DIFFERENTIAL EQUATIONS

given

$y = \phi(x)$; y - a function of x ; where x is an independent variable

$\phi'(x) = \frac{dy}{dx}$; derivative of y ; dependent variable

$y = e^x$; $y' = e^x$; $(\sin(x))' = \cos(x)$; $y' = \frac{dy}{dx}$

DEF : An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a DIFFERENTIAL EQUATION (D.E.)

CLASSIFICATION OF D.E.

a) by type

- ORDINARY D.E. (O.D.E.) $2y' + xy = e^{2x} + 1$
- PARTIAL D.E. (PDE) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

another notation of partial derivatives $u = u(x,y)$

$u_{xx} + u_{yy} = 0$

(b) by ORDER

ex. $f(x) = x^3 + 2x + 7$ – poly. of 3^{rd} degree

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^4 + 7x = e^x$$, 2^{nd} order D.E.

$$M(x,y)\,dx + N(x,y)\,dy = 0$$ – 1^{st} order D.E.
– differential form of 1^{st} order D.E.
– more info need to determine if x or y is the dep. variable

– a n-th order D.E. can be written as a relationship between $n+2$ variables:

$$F(x, y, y', y'', \dots, y^{(n)}) = 0$$

a real-valued function

SAME

$$10x\,y' + 2y + x = 0$$; x – independent var.
y – dependent variable

$a_2(x)$ $a_1(x)$ $a_0(x)$

$$\frac{dy}{dx} = f(x,y) = -\left(\frac{1}{10} + \frac{1}{5x}y\right)$$

$$\frac{d^ny}{dx^n} = f(x, y, y', \dots y^{(n-1)})$$ – normal form

c) by linearity

A n-th order DE is said to be linear if F is linear in $y, y', y'' \ldots, y^{(n)}$. This means that we can write the equation:

$$g(x) = a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y$$

$(1-y)y' + 2y = e^x$;

non-linear ; coef. of y' is not a funct. of "x"