

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	203	All

Examination	Date	Pages
Final	April 2011	3

Instructors	Course Examiners
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Special Instructions

▷ Only Sharp EL 531 or Casio FX 300 MS calculators are allowed.

MARKS

- [11] 1. (a) Sketch the graph of the function $f(x) = |x^2 - 2x|$ starting from the graph of the standard parabola and using appropriate transformations.
- (b) Suppose $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{1+x}$. Find $f \circ g$, $g \circ f$, and $f \circ f$.
- (c) Find the inverse of the function $f(x) = 2 - e^x$. Determine the domain and range of f and f^{-1} .

[8] 2. Evaluate the limits:

(a) $\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$ (b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

Do not use l'Hôpital's rule.

[11] 3. (a) Consider the function $f(x) = \frac{|x - 1|(x + 3)}{x^2 - 1}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

(b) Find parameters a and b such that the function

$$f(x) = \begin{cases} \frac{|x + 1|}{x + 1}, & \text{if } x < -1 \\ ax, & \text{if } -1 \leq x < 1 \\ bx + 2, & \text{if } x \geq 1 \end{cases}$$

will be continuous at every point.

[16] 4. Find derivatives of the functions (you don't have to simplify the answers):

(a) $f(x) = \frac{(\sqrt{x} - 2x)^2 - 3x^2}{x}$;

(b) $f(x) = e^2 + e^{x^2} \ln(1 - 5x)$;

(c) $f(x) = \frac{\sin^3 x}{\cos(2x) + \tan(x)}$;

(d) $f(x) = \arctan(\sqrt{1 + \ln(3x - 1)})$;

(e) $f(x) = (1 + \sin x)^{\tan(x)}$ (use logarithmic differentiation).

[12] 5. Given the function $f(x) = \sqrt{36 - x}$,

(a) Use appropriate differentiation rules to find the derivative of the function.

(b) Derive the same result for $f'(x)$ using the definition of the derivative as the limit of the difference quotient.

(c) Find the linear approximation of $f(x)$ at $a = 0$.

(d) Use this linear approximation to approximate $\sqrt{32}$.

[17] 6. (a) Verify that the point $(2,1)$ belongs to the curve defined implicitly by the equation $x^2 + 2y^2 + 2 = x^3y^3$, and find an equation of the tangent line to the curve at this point.

(b) Two cars are moving away from the intersection of two orthogonal streets at the speeds $v_1 = 15$ m/s going east, and $v_2 = 20$ m/s going north. How fast is the distance between the cars increasing at the instant when the first car is at the distance $x = 30$ m and the second car is at the distance $y = 40$ m from the intersection?

(c) Use l'Hôpital's rule to evaluate $\lim_{x \rightarrow 0} \frac{\sin^2(5x)}{1 - \cos(x)}$.

- [6] 7. Let $f(x) = \sqrt{1 + 4x}$.
- Find the slope m of the secant line joining the points $(0, f(0))$ and $(2, f(2))$.
 - Find all point(s) $x = c$ (if any) on the interval $[0, 2]$ such that $f'(c) = m$.
- [5] 8. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the printed material on the poster is a rectangle with an area of 384 cm^2 , find the dimensions of the poster with the the smallest area.
- [14] 9. Given the function $f(x) = \frac{x^2}{x^2 + 9}$,
- Find the domain and check for symmetry. Find all horizontal and vertical asymptotes (if any).
 - Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).
 - Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).
 - Sketch the graph of the function.

[5] **Bonus Question**

Suppose f and g are both concave upward on $(-\infty, \infty)$. What condition on f is sufficient to ensure that the composite function $h(x) = f(g(x))$ is also concave upward?