Example 15

$$\begin{bmatrix} 1 & 2 & -3 & 3 \\ -2 & -5 & 4 & 5 \\ -5 & -13 & 9 & 18 \end{bmatrix}$$

$\sim$

$R_2 : R_2 + 2R_1$
$R_3 : R_3 + 5R_1$

$$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & -1 & -2 & 11 \\ 0 & -3 & -6 & 33 \end{bmatrix}$$

$\sim$

$-R_2$

$$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & 1 & 2 & -11 \\ 0 & -3 & -6 & 33 \end{bmatrix}$$

$\sim$

$R_3 : R_3 + 3R_2$

$$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & 1 & 2 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- System is consistent
- Infinite Solutions

To make RREF

$\sim$

$R_1 : R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -7 & 25 \\ 0 & 1 & 2 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\downarrow$

$$x - 7z = 25$$
$$y + 2z = -11$$

Example 14.

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & -1 & 1 & 1 \\ -2 & -4 & -2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -3 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 : R_2 - R_1$
$R_3 : R_3 + 2R_1$

No solution.

Example 16

$$\begin{aligned} x + 2y - z &= 1 \\ -2x - 3y + 2z &= -1 \\ -5x - 8y + 5z &= k. \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & -2 & 1 \\ -2 & -3 & 2 & -1 \\ -5 & -8 & 5 & k \end{bmatrix}$$

$\sim$
$R_2 : R_2 + 2R_1$
$R_3 : R_3 + 5R_1$

$$\begin{bmatrix} 1 & 2 & -2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & k+5 \end{bmatrix}$$

$\sim$
$R_3 : R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & -2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & k+3 \end{bmatrix}$$

1. No solution. and $k \neq -3$.
2. ∞ Solution $k = -3$.

1.3 Vector Equations.

Vectors -> A matrix with only one column.

$$\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 0 \\ + \\ -3 \\ 4 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

- Vectors are co-ordinates (with magnitude and direction)

Vectors in R^2 - 2×1 matrix.

R^2 = 2 Spaces

- Set of all vectors with 2 real number entries
- Set of all ordered pairs of real numbers

$$= \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in R \right\}$$

Notations

$$\begin{bmatrix} x \\ y \end{bmatrix} = [x, y]$$

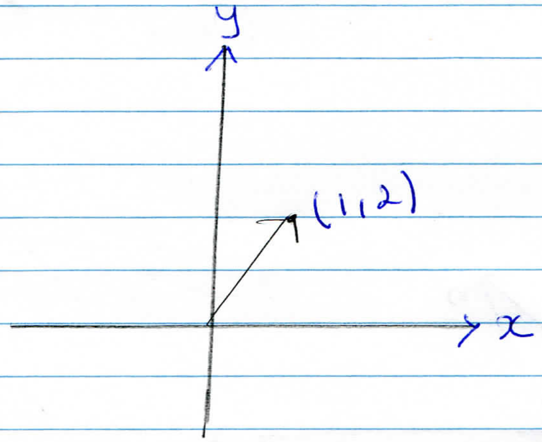
$$= (x, y)$$

$$= \langle x, y \rangle$$

$$\neq [x \;\; y]$$

Zero vector $(\vec{0}) = [0, 0]$

Vector $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ x, y plane (Position vector)



Vector Operations in R^2

1 Addition
- Triangle Rule.
- Parallelogram rule
-

Algebraically:

$\vec{a} = [a_1, a_2]$ $\vec{b} = [b_1, b_2]$

$\cancel{\vec{a} + \vec{b} = [(a_1 + a_2), (b_1 + b_2)]}$

$\vec{a} + \vec{b} = [(a_1 + b_1), (b_1 + b_2)]$

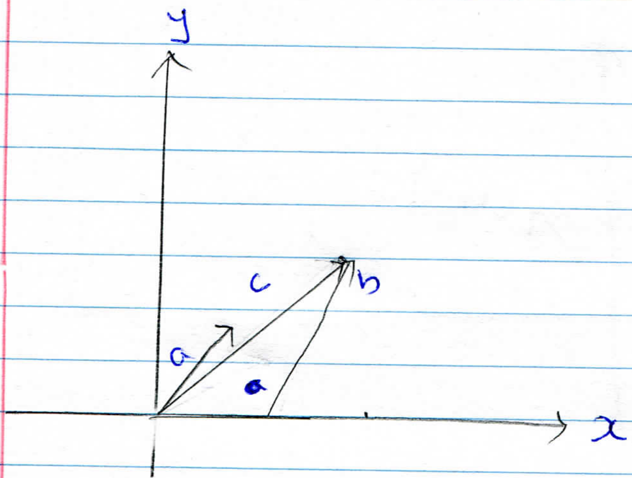
$= \vec{a} = [1, 2]$ $\vec{b} = [3, 4]$

$\vec{a} + \vec{b} = [(1+3), (2+4)]$

$= [4, 6]$

$\vec{a} + \vec{b}$

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$



2 Vectors are scalar multiples of each other if and only if they are parallel.

Algebraically

$\vec{a} = [a_1, a_2]$

$c\vec{a} = [ca_1, ca_2]$

1 $\vec{v} = [1, -2]$

$3\vec{v} = [3, -6]$ or $\begin{bmatrix} 3 \\ -6 \end{bmatrix}$

2 $-V = -\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

3 $0\vec{v} = 0\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Vectors in R^3

R^3 - 3 spaces ~~3~~ (3×1 matrix) $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Vectors in n - space.

n = positive integer.

Let $\vec{a}, \vec{b}, \vec{c} \in R^n$

a ~~S~~. $3\vec{a} + (5\vec{b} - 2\vec{a}) + 2(\vec{b} - \vec{a})$.

$3\vec{a} + (5\vec{b} - 2\vec{a}) + (2\vec{b} - 2\vec{a})$

$= -\vec{a} + 7\vec{b}$

b. $5\vec{c} - \vec{a} = 2(\vec{a} + 2\vec{c})$

$5\vec{c} - \vec{a} = 2\vec{a} + 4\vec{c}$

$\vec{c} = 3\vec{a}$

$= \vec{c} - 3\vec{a}$

$\boxed{\vec{a} = \frac{\vec{c}}{3}}$

Linear Combination.

E

1 Determine if vector $\vec{x} = [11, -6]$ is a linear combination of vectors $\vec{y} = [1, 3]$ and $\vec{z} = [5, 2]$. If so give the coefficient

$$a\vec{y} + b\vec{z} = \vec{x}$$

$$a\begin{bmatrix} 1 \\ 3 \end{bmatrix} + b\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ -6 \end{bmatrix}$$

$$a + 5b = 11$$
$$3a + 2b = -6$$

$$\begin{bmatrix} 1 & 5 & 11 \\ 3 & 2 & -6 \end{bmatrix}$$

$$\sim$$

$$R_2 : R_2 - 3(R_1) \qquad \begin{bmatrix} 1 & 5 & 11 \\ 0 & -13 & -39 \end{bmatrix}$$

$$-13b = -39$$
$$b = 3.$$

$$a + 5(3) = 11$$
$$a = -4.$$

$$\boxed{\begin{aligned} a &= -4 \\ b &= 3 \end{aligned}}$$