Brief solutions to Term Test 1

Problem 1. : $5, \quad 5, \quad (1,1,1,1)^T, \quad (0,1,0,0)^T.$

Problem 2. : $\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/3 & 1 & 0 & 0 \\ 0 & 3/8 & 1 & 0 \\ 0 & 0 & 8/21 & 1 \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 8/3 & 1 & 0 \\ 0 & 0 & 21/8 & 1 \\ 0 & 0 & 0 & 55/21 \end{pmatrix},$

$\mathbf{g} = (4\ ,\ 11/3\ ,\ 13/8\ ,\ -55/21)^T, \quad x = (1,1,1,-1)^T.$

Problem 3. : $\mathbf{T} = \mathbf{D}[\mathbf{I} + \mathbf{B}]$, where $\| \mathbf{B} \|_\infty = 5/6 < 1.$

$\| T_n^{-1} \|_\infty \ \leq \ 1\ , \quad \mathrm{cond}(T_n) \ \leq \ 5\ .$

Problem 4. $n-1$ divisions only.

Problem 5.

$$x^{(k+1)} \ = \ \frac{2(x^{(k)})^3 + 2}{3(x^{(k)})^2}, \qquad x^{(1)} = 4/3, \qquad x^{(2)} = 91/72, \qquad x^{(3)} = 1.25993.$$

Problem 6. $x_1^{(1)} = 1/2, \qquad x_2^{(1)} = 7/8.$

Problem 7. $x^{(1)} = 8/9, \quad x^{(2)} = 32/81.$

Fixed points: $x^* = 0$ (repelling), and $x^* = 3/4$ (repelling).
Quadratic convergence: NA (not applicable).

Problem 8. Converges to the unique fixed point $c \cong 0.73908$.