COMP. 233.

ASSIGN. 3.

1. A certain small freight elevator has a max. capacity C, which is Normally distributed, with mean 400 lbs., and standard deviation 4 lbs.

   The weight of the boxes being loaded into the elevator is a R.V., with mean 30 lbs., and standard deviation 0.3 lbs.

   How many boxes may be loaded into this elevator before the probability of disaster exceeds 0.2?

   Consider the random variable, C, to be independent of the weight of the boxes.

2. According to the National Transportation Safety Board, the number of people/car passing through a certain intersection between 8:00 – 9:00 A.M., is a R.V., H (for Humans), with mean 4 people, and variance 2 people.

   If a random sample of 30 cars is chosen, at this intersection, during this time period, what is the probability that the average number of people/car will be at least 5?

   Assume this Study was performed during business days (weekdays).

3. The Scholastic Aptitude Test, Mathematics test scores across the population of high school seniors follow a normal distribution with mean 500 and standard deviation 100.

   If five seniors are randomly chosen, find the probability that:

   (a) All scored below 600.
   (b) Exactly three of them scored above 640.

4. The annual snowfall in Chicoutimi is normally distributed with mean 3.14 m. and standard deviation 60 cm.

   (a) What is the probability this years snowfall will exceed 3.50 m.?
   (b) What is the probability that the sum of the next 2 years snowfall will exceed 7.00 m.?
   (c) What is the probability that the sum of the next 3 years snowfall will exceed 10.5 m.?

5. The following are the percentages of ash content in 12 samples of coal found in close proximity:

   9.2; 14.1; 9.8; 12.4; 16.0; 12.6; 22.7; 18.9; 21.0; 14.5; 20.4; 16.9

Find the:

(a) Sample mean.
(b) Sample median.
(c) Sample standard deviation.

6. If 10 pairs of fair dice are rolled, what is the probability that the sum of the values obtained is between 30 and 40 inclusive.

7. The age at which the fan assembly in a new laptop fails is normally distributed with variance σ^2. If seven new laptops are used, find:

   (a) $P\left(\frac{S^2}{\sigma^2} \leq 2\right)$.

   (b) $P\left(\frac{1}{6} \leq \frac{S^2}{\sigma^2} \leq \frac{7}{3}\right)$.

   S^2 is the sample variance of the seven data values.

COMP. 233.

Assign. 3 Solutions.

1. Let R.V. X_i: denote the weight of the i th. box being loaded into the elevator $i = 1, 2, 3, \ldots\ldots$

P_n: Prob. that weight of n loaded boxes exceeds capacity, C. (C is max. capacity).

$$X = \sum_{i=1}^{n} X_i.$$

Now then;

$$P_n = P(X_1 + X_2 + X_3 + \ldots\ldots + X_n > C)$$

$$= P\left(\sum_{i=1}^{n} X_i > C\right)$$

$$P_n = P\left(\sum_{i=1}^{n} X_i - C > 0\right).$$

By the C.L.T. (Central Limit Theorem) $\Rightarrow$

$\sum_{i=1}^{n} X_i$ is Normally Dist. with mean $30n$, and variance $.09n$.

Given: mean of C is 400, variance is 16.

$$E\left[\sum_{i=1}^{n} X_i - C\right] = E[X - C]$$

$$= E[X] - E[C]$$

$$= 30n - 400.$$

$$Var(X - C) = Var(X) + Var(C)$$

$$= .09n + 16.$$

Standardizing:

$$P_n = P\left(\sum_{i=1}^{n} X_i - C > 0\right)$$

$$= P(X - C > 0)$$

$$= P\left(\frac{(X-C) - (30n - 400)}{\sqrt{.09n+16}} > \frac{-(30n - 400)}{\sqrt{.09n+16}}\right)$$

$$P_n = P\left(Z > \frac{-(30n - 400)}{\sqrt{.09n+16}}\right).$$

What is sought is the max. $n \ni P_n \leq .2$;

OR

$$P\left(Z > \frac{400 - 30n}{\sqrt{.09n+16}}\right) \leq .2.$$

$P(Z \geq .84) \approx .2$ (Look in table)

$\therefore$,

$$\frac{400 - 30n}{\sqrt{.09n + 16}} \geq .84$$

$$400 - 30n \geq .84\sqrt{.09n + 16}$$

$n \leq 13$ boxes.

2. Let R.V. X_i : denote the number of people in the i th. car. $i = 1, 2, 3, \ldots, 30$.

$$\bar{X} = \sum_{i=1}^{30} \frac{X_i}{n}.$$

What is sought is the prob. that the average number of people/car ≥ 5; cars chosen from a random sample.

$$P\left(\frac{X_1 + X_2 + X_3 + \cdots + X_{30}}{30} \geq 5\right)$$

$$= P\left(\sum_{i=1}^{30} \frac{X_i}{30} \geq 5\right)$$

$$= P(\bar{X} \geq 5).$$

By the C.L.T. $\bar{X} \approx N\left(4, \frac{\sqrt{2}}{\sqrt{30}}\right)$. $\sigma = \frac{\sqrt{2}}{\sqrt{30}}$.

Standardizing:

$$P\left(\frac{\bar{X} - 4}{\sqrt{1/15}} \geq \frac{5-4}{\sqrt{1/15}}\right) = P\left(Z \geq \frac{1}{\sqrt{1/15}}\right)$$

$$= P(Z \geq \sqrt{15})$$

$$= P(Z \geq 3.87) \doteq 0.$$

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Assign. 3 Solutions - Problems 3-7.

3. Let R.V. X: a senior student test score.

A. Probability of one student scoring below 600 is:

$P(X<600)$?

$$P(X<600) = P\left(\frac{X-500}{100} < \frac{600-500}{100}\right)$$

$$P(X<600) = P(Z<1) = .8413.$$

$\therefore$ For the five students, the prob. that all scored below 600 is:

$$[P(X<600)]^5 = (.8413)^5 = .4214.$$

B. Probability of one student scoring above 640 is:

$P(X>640)$?

$$P(X>640) = P\left(\frac{X-500}{100} > \frac{640-500}{100}\right)$$

$$= P(Z>1.4)$$

$$P(X>640) = 1-P(Z\le 1.4) = 1-.9292 = .0808.$$

$\therefore$ Probability of exactly 3 students scoring above 640 is:

$$B(3;5,.0808) = \binom{5}{3}(.0808)^3(.9292)^2 = .0045.$$

4. Let R.V. X: be the annual snowfall.

A. $P(X > 3.5)$?

$$P(X > 3.5) = P\left(\frac{X-3.14}{.6} > \frac{3.5-3.14}{.6}\right)$$

$$= P(Z > .6)$$
$$= 1 - P(Z \le .6)$$
$$= 1 - .7257$$
$$P(X > 3.5) = .2743.$$

B. Let R.V. X_1: denote snowfall of 1st. year.
X_2: " " " 2nd. year.

$$X = X_1 + X_2.$$

By the C.L.T., $X \sim N(3.14(2), .6\sqrt{2}) = N(\mu, \sigma)$.

$$\therefore P(X > 7) = P\left(\frac{X-6.28}{.6\sqrt{2}} > \frac{7-6.28}{.6\sqrt{2}}\right)$$

$$= P(Z > .85)$$
$$P(X > 7) = 1 - P(Z \le .85) = 1 - .8023 = .1977.$$

C. Let R.V. $\left.\begin{matrix} X_1 \\ X_2 \end{matrix}\right\}$ as in part B.

X_3 : snowfall in 3rd. year.

$$X = X_1 + X_2 + X_3.$$

$$X \sim N(\mu, \sigma) = N(3.14(3), .6\sqrt{3}).$$

$$P(X > 10.5) = P\left(\frac{X - 9.42}{.6\sqrt{3}} > \frac{10.5 - 9.42}{.6\sqrt{3}}\right)$$

$$= P(Z > 1.04)$$
$$= 1 - P(Z \leq 1.04)$$
$$= 1 - .8508$$
$$P(X > 10.5) = .1492.$$

5. A. $n = 12$; $\sum_{i=1}^{12} X_i = 188.5$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{12}\sum_{i=1}^{12} X_i = \frac{188.5}{12} \approx 15.71$$

where X_i : i th. percentage.

B. Data must be sorted (in ascending order).

Let R.V. X_i : as in part A (after data is sorted).

$$\text{median} = \begin{cases} X_{\lceil \frac{n}{2} \rceil} & \text{if } n \text{ is odd} \\ \frac{1}{2}\left(X_{(\frac{n}{2})} + X_{(\frac{n}{2}+1)}\right) & \text{if } n \text{ is even.} \end{cases}$$

Since $n = 12$, even $\Rightarrow$

$$\text{median} = \frac{(X_6 + X_7)}{2} = \frac{1}{2}(14.5 + 16) = 15.25.$$

c. Let S^2: be the sample variance.

$$S^2 = \frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{n-1}.$$

$n = 12$; $\bar{X} = 15.71$; $\sum_{i=1}^{12} X_i^2 = 3{,}173.53.$

$$\therefore S^2 = \frac{3{,}173.53 - (12)(15.71)^2}{11} = 19.26 \Rightarrow$$

$$S = \sqrt{19.26} \doteq 4.4.$$

6. Let R.V. X_i: outcome of the roll of the ith. die.

$$X = \sum_{i=1}^{20} X_i.$$

$$E[X] = E\left[\sum_{i=1}^{20} X_i\right] = \sum_{i=1}^{20} E[X_i] = 3.5(20).$$

$$Var(X) = Var\left(\sum_{i=1}^{20} X_i\right) = \sum_{i=1}^{20} Var(X_i) = 20\left(\frac{35}{12}\right).$$

$$\sigma_X = \sqrt{Var(X)} = \sqrt{20}\sqrt{\frac{35}{12}} = 7.64.$$

Since X is the sum of 20 independent and identically distributed R.V.'s, each with mean $= 3.5$, and variance $= \frac{35}{12}$ $\Rightarrow$ by the C.L.T. that $X \sim N\left(20(3.5), \sqrt{20}\sqrt{\frac{35}{12}}\right) = N(\mu, \sigma)$.

$P(30 \le X \le 40)$?

$$P(30 \le X \le 40) = P\left(\frac{30 - 20(3.5)}{7.64} \le \frac{X - 20(3.5)}{7.64} \le \frac{40 - 20(3.5)}{7.64}\right)$$

$$= P(-5.24 \le Z \le -3.93)$$

$$= [1 - P(Z \le 3.93)] - [1 - P(Z \le 5.24)]$$

$$\approx 0.$$

7\. A. $P\left(\frac{S^2}{\sigma^2} \le 2\right)$?

$$P\left(\frac{S^2}{\sigma^2} \le 2\right) = P\left(\frac{(n-1)S^2}{\sigma^2} \le (n-1)2\right)$$

$$= P\left(\chi^2_{(n-1)} \le (n-1)2\right).$$

Since there are seven laptops $\Rightarrow n=7$

$\therefore$ $P\left(\chi^2_{(n-1)} \le (n-1)2\right) = P\left(\chi^2_6 \le 12\right) < .95.$

B. $P\left(\frac{1}{6} \le \frac{S^2}{\sigma^2} \le \frac{7}{3}\right)$?

$$P\left(\frac{1}{6} \le \frac{S^2}{\sigma^2} \le \frac{7}{3}\right) = P\left(\frac{n-1}{6} \le \frac{(n-1)S^2}{\sigma^2} \le \frac{(n-1)7}{3}\right)$$

$$= P\left(\frac{n-1}{6} \le \chi^2_{(n-1)} \le (n-1)\frac{7}{3}\right)$$

$n=7$:

$$= P(1 \le \chi^2_6 \le 14).$$

$P(\chi^2_6 \le 14) < .975$; $P(\chi^2_6 \le 1) < .025.$

$\therefore$ $P(\chi^2_6 \le 14) - P(\chi^2_6 \le 1) < .975 - .025 = .950.$