Solution to Test 1(B)

MAY1320B, Fall 2015
Total = 20 marks

Part I. Short Answer Questions: (14 marks)

Only the final answer is to be marked.

1. (2 marks) Assume some values of a one-to-one function $y = f(x)$ are given in the following table

x	1	2	3	4	5
$f(x)$	4	5	1	2	3
$f'(x)$	2	5	1	3	4

Let $g = f \circ f$, and $h = f^{-1}$.

(a) $g(4) =$ ______________________ .

(b) $h(4) =$ ______________________ .

Solution. (a) $g(4) = f(f(4)) = f(2) = 5$.

(b) Since $f(1) = 4$, $f^{-1}(4) = 1$.

2. (2 marks) The limit $\lim\limits_{x\to-1} \dfrac{x^2-1}{2x^2-x-3} =$ ____________________ .

Solution. $\lim\limits_{x\to-1} \dfrac{x^2-1}{2x^2-x-3} = \lim\limits_{x\to-1} \dfrac{(x-1)(x+1)}{(2x-3)(x+1)} = \lim\limits_{x\to-1} \dfrac{x-1}{2x-3} = \dfrac{-2}{-5} = \dfrac{2}{5}$.

3. (2 marks) If a tangent line of the graph of function $y = x^2$ is parallel to the line $4x + y = 1$. The equation of the tangent line is

$y =$ ______________________________ .

Solution. The slope of the given line is −4. $y' = 2x = -4$. Then $x = -2$. $y = 4$. The equation of the tangent line is $y = -4(x + 2) + 4$, or $y = -4x - 4$.

4. (2 marks) Let $f(x) = \dfrac{x}{e^x}$. $f'(x) =$ ___________________ .

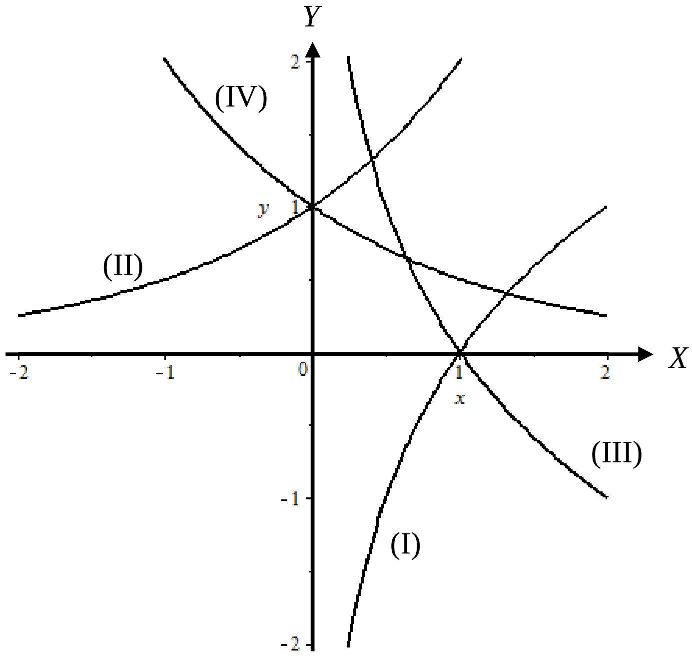
Solution. $f'(x) = \dfrac{e^x - xe^x}{e^{2x}} = \dfrac{e^x(1-x)}{e^{2x}} = \dfrac{1-x}{e^x}$.

5. (2 marks) Solve equation $\log_2(x + 3) - \log_2(5x + 3) = -1$. Then x = ______________ .

Solution. $\log_2 \frac{x+3}{5x+3} = -1$, $\frac{x+3}{5x+3} = \frac{1}{2}$, $2x+6 = 5x+3$, $x = 1$.

6. (2 marks) Match the following graphs with functions

(a) $y = \log_2 x$, (b) $y = \log_{(1/2)} x$, (c) $y = 2^x$, (d) $y = \left(\frac{1}{2}\right)^x$.



Answer. (I) -> (a), (II) -> (c), (III) -> (b), (IV) -> (d).

7. (2 marks) The range of function $y = \arccos x$ is __________________________ .

Answer. $0 \le y \le \pi$.

Part II. Detailed Answer Question

1. (3 marks) **Use the definition of the derivative** to find the derivative of the function $y = \frac{1}{\sqrt{x}}$.

Solution. $y' = \lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)=\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x(x+h)}}=\lim_{h\to 0}\frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{h\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}$

$=-\lim_{h\to 0}\frac{h}{h\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}=-\lim_{h\to 0}\frac{1}{\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}=-\frac{1}{2x\sqrt{x}}$.

2. (3 marks) Find all vertical and horizontal asymptotes of the graph of the function
$y = \frac{3x^2-1}{x^2-x-2}$.

Solution. Let $x^2 - x - 2 = 0$, $x = -1, 2$. This function has vertical asymptotes $x = -1$, and $x = 2$.

$\lim_{x\to\infty}\frac{3x^2-1}{x^2-x-2} = \lim_{x\to-\infty}\frac{3x^2-1}{x^2-x-2}=3$. This function has horizontal asymptote $y = 3$.