FINAL EXAM-MATH 1339 C
FALL TERM, 2014

Professor: Camelia Karimianpour

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First Name:_____**Last Name:**_____

I.D. Number_____

Instructions-This final examination consists of 12 multiple choice questions worth 1 point each. Your answers to the multiple choice questions must be clearly marked in the squares below. There are also 7 long answer questions. The test is worth 50 marks in total. For the long answer questions, you must show your work **on the exam itself** and clearly display your answers. **Do not unstaple these pages.**

ONLY BASIC CALCULATORS. NO BOOKS. NO NOTES.

Multiple Choice Answers:

☐ #1	☐ #2	☐ #3	☐ #4
☐ #5	☐ #6	☐ #7	☐ #8
☐ #9	☐ #10	☐ #11	☐ #12

Question 1- Let $f(x) = 2x^2 + 3x + 1$. Determine the average rate of change of f over the interval $[-3, 2]$.

A) 1 **B)** 5 **C)** $\frac{-1}{3}$ **D)** $\frac{-1}{2}$ **E)** none of the above

Question 2- The vector $\vec{v}$ and $-5\vec{v}$ are

A) parallel
B) in opposite directions
C) collinear
D) all of the above
E) none of the above

Question 3- Suppose $g(x) = x\sin(\pi x)$. Find the equation of the tangent line at $x = 1/2$.

A) $y = x$ **B)** $y = -x$ **C)** $y = x - 1/2$ **D)** $y = -x - 1/2$ **E)** $y = 2x + 1$

Question 4- Find the following limit:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

A) 1 **B)** 4 **C)** 2 **D)** 0 **E)** the limit does not exist

Question 5- Find $f'(e)$ when $f(x) = (\ln x)^2$.

A) 1 **B)** $\frac{2}{e}$ **C)** 2 **D)** e **E)** none of these answers

Question 6- Let $\vec{u} = [2, -1, 3]$ and $\vec{v} = [-1, -2, 0]$. Find $(-2\vec{u} + \vec{v}) \cdot \vec{u}$.

A) -18
B) 18
C) -28
D) 0
E) none of the above

Question 7- Determine the area of the parallelogram defined by the vectors $\vec{u} = [1, 2, 3]$ and $\vec{d} = [-2, 1, -1]$.

A) $\sqrt{75}$ **B)** $-\sqrt{75}$ **C)** 5 **D)** 25 **E)** none of the answers

Question 8- Determine the projection of $\vec{v} = [2, -4]$ on $\vec{u} = [1, 3]$.

A) $[-1/3, -1]$ **B)** $[-1, -3]$ **C)** $[0, 0]$ **D)** $[2, 3]$ **E)** none of the answers

Question 9- What are the critical points of the function $f(x) = xe^x$?

A) 0 and 1 **B)** e **C)** 0 **D)** -1 **E)** none of the answers

Question 10- Given the vector $\vec{u} = [-2, 4, 2]$, which of the following vectors is perpendicular to $\vec{u}$?

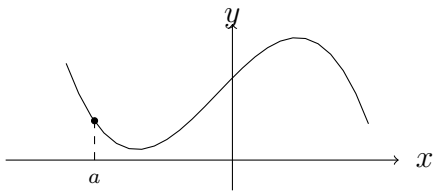
A) $\vec{u'} = [1, 0, 0]$
B) $\vec{u'} = [1, 1, 0]$
C) $\vec{u'} = [5, 1, 3]$
D) $\vec{u'} = 3[-2, 4, 2]$
E) none of the answers above.

Question 11- Suppose that a function f satisfies $f'(a) = 0$ and $f''(a) > 0$. Then which of the following is true?

A) f has a local minimum at $x = a$.
B) f has a local maximum at $x = a$.
C) f has an inflexion point at $x = a$.
D) the line tangent to the graph of f at the point (a,f(a)) is a vertical line
E) none of the above.

Question 12- Consider the graph of $y = f(x)$. Which statement is true?

A) $f'(a) > 0$ and $f''(a) > 0$
B) $f'(a) > 0$ and $f''(a) < 0$
C) $f'(a) < 0$ and $f''(a) > 0$
D) $f'(a) < 0$ and $f''(a) < 0$
E) none of the answers



Long Answer Question 1 (4 points)
Consider the following function:

$$f(x) = \begin{cases} 1 & \text{if } x < -2 \\ \frac{x^2+1}{x-3} & \text{if } -2 \leq x \leq 0 \\ \ln x & \text{if } 0 < x < e \\ 1 & \text{if } x \geq e. \end{cases}$$

Determine all the discontinuities of the function and their type. You should write your solution completely and justify your answers.

Long Answer Question 2 (6 points)
Find the derivatives of the following functions. Do not simplify.

(a) $f(x) = \sin(x^2) + \dfrac{5}{x} - e^{3x} + 4x^3$

(b) $g(x) = e^{\cos(x)}(3x^2 - 5x)$

(c) $h(x) = \dfrac{(x^2+3)^{100}}{e^x}$

Long Answer Question 3 (8 points)
Consider the function

$$f(x) = \frac{3x}{x^2 - 9}$$

(a) Determine the domain of $f(x)$.

(b) Find the horizontal and vertical asymptotes of $f(x)$.

(c) Find the x-intercept and y-intercept of $f(x)$.

(d) Find the critical points and the intervals where $f(x)$ is increasing or decreasing. Find all the local extrema.

(e) Find the points of inflection and the intervals where $f(x)$ is concave up or down.

(f) Sketch the graph.

Long Answer Question 4 (6 points)

(a) **(4 points)** Determine the vector and parametric equations of the line (L1) passing through points A$(-3, 2, 8)$ and B$(4, 3, 9)$.

(b) **(3 points)** Determine if the lines L1 (from part a) and L2: $[x, y, z] = [1, 0, -1]+s[14, 2, 2]$ intersect. Why or why not?

(c) **(3 points)** Is the point $(4, 3, 3)$ on the line L1? Why or why not?

Long Answer Question 5 (4 points)
Consider the plane passing through points A$(1, 2, 3)$ and B$(-1, 2, 1)$ and C$(3, 1, 4)$.

(a) Write the vector equation of the plane.

(b) Write the scalar equation of the plane.

Long Answer Question 6 (5 points)
A company manufacturing hats has a demand (or price) function $p(x) = -0.025x + 49.5$, where x is the number of hats and $p(x)$ is the price of a hat, and a cost function

$$C(x) = -0.0005x^2 + 7.5x + 200.$$

(i) Find the revenue function and the profit function.

(ii) Determine the marginal cost when $x = 1800$.

(iii) How many hats should be produced and sold to maximize profits? Justify your answer.

Long Answer Question 7 (5 points) Let $\vec{u} = [1, 1, 1]$, $\vec{v} = [2, 2, 4]$.

(a) Find $\vec{u} \cdot \vec{v}$.

(b) Find $\vec{u} \times \vec{v}$.

(a) Show that $\vec{u} \times \vec{v}$ is perpendicular to $\vec{v}$.

Extra page for additional work

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