MAT 2371
Final Examination

December 2013 **Professor G. Ivanoff**
Time: 3 hours

Student Number:________________ **Seat number:**____________

Family Name: ________________ **First Name:** ____________

- **This is an open book examination. Calculators are the only electronic device permitted.**
- **The exam consists of two parts: A (multiple choice) and B (long answer). For part A, record your answer to each question in the table provided on the next page. For part B, write your answers in the space provided on the questionnaire.**
- **At the end of the examination, hand in the entire questionnaire.**

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Professor's use only:

Part A		
Part B	1	
	2	
	3	
	4	
TOTAL		

PART A: Multiple choice. 4 marks per question.

Put the letter corresponding to the correct answer in the space provided in the table below.

Question	1	2	3	4	5	6	7	8
Answer								
Question	9	10	11	12	13	14	15	Total
Answer								

1. Suppose that 23% of Canadian adults have a pet. Find the probability that among 3 Canadian adults chosen at random, at least one has a pet.

   (A) 0.2300 (B) 0.6900 (C) 0.4091 (D) 0.5435 (E) 0.5909

2. Suppose that the number of spiders in my basement follows a Poisson process with rate 0.3 spiders per 100 square feet. The area of my basement is 600 square feet. What is the probability that I will find at least 2 but no more than 4 spiders the next time I vacuum my basement floor?

   (A) 0.269 (B) 0.501 (C) 0.234 (D) 0.037 (E) 0.569

3. Let X be the number of smokers in a married couple. X is a random variable that can take on the values $0, 1, 2$ with probabilities p_0, p_1, p_2. If it is known that the mean of X is 0.53, and that there is at least one smoker in 30% of married couples, find the probabilities p_0, p_1, p_2.

   (A) $p_0 = 0.7; p_1 = 0.07; p_2 = 0.23$ (B) $p_0 = 0.47; p_1 = 0.53; p_2 = 0$
   (C) $p_0 = 0.7; p_1 = 0.15; p_2 = 0.15$ (D) $p_0 = 0.7; p_1 = 0; p_2 = 0.3$
   (E) $p_0 = 0.67; p_1 = 0.13; p_2 = 0.2$

4. My birdfeeders attract squirrels as well as birds. In an attempt to discourage the squirrels, I have put safflower seeds in one of my feeders and sunflower seeds in the other. I have observed the number of visits of the various types of birds (and squirrels) using my feeders and also the proportion of each type that choose safflower seeds instead of sunflower seeds. The data is given below:

Animal	% of visits to feeders	Proportion choosing safflower
Cardinal	15	0.4
Chickadee	40	0.3
Finch	20	0.2
Squirrel	25	0.1

Find the probability that the next animal (bird or squirrel) to visit my feeders will choose safflower seeds.

(A) 0.9 (B) 1 (C) 0.245 (D) 0.22 (E) 0.195

5. Referring to question #4, I left some safflower seeds close to my window, hoping to get a photo of a cardinal. Unfortunately, while I was looking for my camera, the seeds were eaten. What is the probability that it was NOT a cardinal who ate the safflower seeds?

(A) 0.9400 (B) 0.6000 (C) 0.8500 (D) 0.7551 (E) 0.1850

6. Let X be a negative binomial random variable with parameters $r = 5$ and $p = \frac{3}{4}$. Find $E[3(X^2)]$.

(A) 400/3 (B) 20 (C) 140 (D) 420 (E) 20/3

7. In the game of Yahtzee, five fair (six-sided) dice are rolled. What is the probability of getting a "full house"? A full house means that we have a pair (2 dice with the same value) and a triple (3 dice with the same value, different from that of the pair).

(A) $\frac{25}{6^5}$ (B) $\frac{20}{6^4}$ (C) $\frac{5}{6^4}$ (D) $\frac{120}{6^5}$ (E) $\frac{50}{6^4}$

8. It is known that 20% of the computer chips produced by a manufacturer are defective. The Faculty of Science has just purchased 36 new computers, each containing a chip produced this manufacturer. Approximately what is the probability that at most 8 of the computers contain a defective chip?

   (A) 0 (B) 0.3707 (C) 0.6293 (D) 0.2946 (E) 0.7054

9. Suppose that X is a normal random variable with mean 5. What is σ, the standard deviation of X, if $P(X > 9) = .025$?

   (A) 4.16 (B) 5.91 (C) 2.04 (D) 2.43 (E) 1.00

10. Future Shop sells a handheld electronic game powered by a single battery that has an exponential lifetime with a mean of 5 hours. If I purchase 4 batteries for my game, what is the probability that my total playing time (the total life of my batteries) is at least 25 hours?

    (A) 0.0571 (B) 0.9429 (C) 0.3085 (D) 0.2650 (E) 0.7350

11. Consider the situation in problem #10. What is the probability that exactly two of my four batteries last longer than 5 hours?

    (A) $2e^{-1}$ (B) $6e^{-2}(1-e^{-1})^2$ (C) e^{-2} (D) $e^{-2}(1-e^{-1})^2$ (E) $\frac{6}{2^4}$

12. Consider the situation in problem #10. If I purchase 40 batteries for my game, approximately what is the probability that my total playing time is at least 250 hours?

    (A) 0.0571 (B) 0.9429 (C) 0.3085 (D) 0.2650 (E) 0.7350

13. Let X be a random variable with probability density function

$$f(x) = 2x, \ 0 < x < 1.$$

Find $P\left(X < \frac{3}{4} \mid X > \frac{1}{4}\right)$.

(A) $\frac{8}{15}$ (B) $\frac{9}{15}$ (C) $\frac{2}{7}$ (D) $\frac{3}{7}$ (E) $\frac{9}{16}$

14. Let X_1, X_2, X_3 be independent and identically distributed with the probability density function in $\#13$. If $Y = \max(X_1, X_2, X_3)$, what is the probability that $Y \leq \frac{1}{2}$?

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{64}$ (E) $\frac{1}{16}$

15. Suppose that X has the $\chi^2(1)$ distribution. Find $\pi_{0.8}$, the 80^{th} percentile of the distribution of X.

(A) 1.282 (B) 1.132 (C) 1.644 (D) 0.842 (E) 0.709

Part B: Long answer 10 marks per question

Write your answers directly on the questionnaire. Clearly define your notation and show all of your calculations.

1. Let X be a continuous random variable with probability density function

$$f(x) = \frac{c}{x^4},\ 1 < x < \infty,$$

where c is a constant.

(a) Find the constant c.

(b) Find the cumulative distribution function of X.

(c) Find the mean and variance of X.

(d) Find the three quartiles of the distribution of X.

2. Suppose that the time taken for my bus ride to the university each morning follows a gamma distribution with $\alpha = 31$ and $\theta = 1.5$. In order to "improve" its service, OC Transpo has decided to change the route that my bus takes to the university. The first morning that I take the new and "improved" route, my trip takes 74 minutes.

   (a) When I complained, OC transpo claimed that the distribution of the time taken for my trip has not changed, and in particular, the mean and variance of the time are still the same. If this is the case, find an upper bound for the probability that the *absolute value* of the difference between the time taken for my next trip and the mean is at least as large as it was today.

(b) After 25 trips on the new route, my average travel time is 50 minutes. If it is true that the distribution has remained unchanged, approximately what is the probability that my average travel time over the next 25 trips is at least 50 minutes?

3. (a) Let X be a continuous random variable with a uniform distribution on the interval $[-1, 1]$. Find the probability density function f_Y of

$$Y = 1 - X^4.$$

(b) Let X be a discrete random variable with probability mass function

$$f_X(x) = \frac{x+5}{45} \quad x \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}.$$

Find the probability mass function of

$$Y = 16 - X^2.$$

4. Consider two independent discrete random variables X and Y with the following probability mass functions:

$$f_X(x) = \frac{1}{3},\ x = -1, 0, 1 \text{ and } f_Y(y) = \frac{1}{2},\ y = 2, 4.$$

Let $W = 2X + Y$.

(a) Find the probability mass function of W using the probability mass functions of X and Y.

(b) Find the moment generating function of W using the moment generating functions of X and Y. Find the probability mass function of W using its moment generating function. Does your answer agree with (a)?