CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2013	2
Instructors:	G. Bobos, J. Brody, A Iovita, H. Proppe	**Course Examiners** A. Atoyan & H. Proppe
Special Instructions:	Only calculators approved by the Department are allowed	

MARKS

[10] **1.** (a) Sketch the graph of the function

$$f(x) = \begin{cases} -\sqrt{9-x^2} & \text{if } -3 \le x < 0 \\ 1 - |x-4| & \text{if } 0 \le x \le 5 \end{cases}$$

on the interval $-3 \le x \le 5$ and calculate the definite integral $\int_{-3}^{5} f(x)\,dx$ as the signed area between the graph of f and the x-axis (do not antidifferentiate).

(b) Use the Fundamental Theorem of Calculus to find a function $f(x)$ and a number a so that $a + \int_4^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}$ for all $x > 0$.

[11] **2.** Calculate the following indefinite integrals:

(a) $\displaystyle\int \frac{(x + x^{3/2})^2}{\sqrt{x}}\,dx$ (b) $\displaystyle\int \frac{5x-3}{x^2-2x-3}\,dx$

[10] **3.** Find the antiderivative $F(t)$ of the function $f(t)$ that satisfies the given condition:

(a) $f(t) = (t-1)\left(t^2 - 2t\right)^{10}$, $F(1) = 0$. (b) $f(t) = \sin^3 t \cos^2 t$, $F\left(\frac{\pi}{2}\right) = 10$.

[12] **4.** Evaluate the following definite integrals (give the exact answers):

(a) $\displaystyle\int_1^e x \ln^2 x\,dx$ (b) $\displaystyle\int_{-\pi/4}^{\pi/4} \tan^2 x\,dx$

[8] 5. Evaluate the given improper integral or show that it diverges:

(a) $\displaystyle\int_1^\infty \frac{1}{x(\ln x)^2}\,dx$ (b) $\displaystyle\int_{-1}^0 \frac{2}{x^2-1}\,dx$

[17] 6. (a) Sketch the curves $y = 2 - x^2$ and $y = -x$ and find the area enclosed by these curves.

(b) Sketch the region enclosed by $f(x) = x^2$, the x-axis, the y-axis, and the line $x = 1$. Find the volume of the solid generated by revolving this region about the y - axis.

(c) Find the average value of the function $f(x) = \sqrt{25 - x^2}$ on the interval $[-5, 5]$.

[9] 7. Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

(a) $a_n = \dfrac{\ln(n)}{\ln(2n)}$ (b) $a_n = \dfrac{n}{n+1}\cos(n\pi)$ (c) $a_n = n - \sqrt{n^2 - n}$

[11] 8. Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(a) $\displaystyle\sum_{n=1}^\infty \frac{\arctan n}{n^{1.2}}$ (b) $\displaystyle\sum_{n=1}^\infty (-1)^{n-1} e^{2/n}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{n^5}{5^n}$

[6] 9. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^\infty \frac{(2x-1)^n}{n\,4^n}$

[6] 10. (a) Use the MacLaurin series for e^{-x} to find the MacLaurin series for $f(x) = e^{-x^2}$.

(b) Find an antiderivative $F(x)$ for $f(x) = e^{-x^2}$ expressed as a power series.

[5] **Bonus Question.** Prove that if $a_n > 0$ and $\sum_{n=1}^\infty a_n$ converges, then $\sum_{n=1}^\infty a_n^2$ also converges. However, if the condition $a_n > 0$ is removed, show that this is not necessarily true, i.e. $\sum_{n=1}^\infty a_n^2$ may diverge.