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CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2012	2
Instructors:		**Course Examiners**
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Special Instructions:	Only calculators approved by the Department are allowed. For full marks show all your work.	

[10] **1.** (a) Sketch a graph of the function

$$f(x) = \begin{cases} -\sqrt{4-x^2} & \text{for } |x| \le 2 \\ |x-3|-1 & \text{for } 2 < x \end{cases}$$

on the interval $-2 \le x \le 4$ and calculate the definite integral $\int\limits_{-2}^{4} f(x)\,dx$ in terms of signed area (*do not* antidifferentiate).

(b) Find the derivative $F'(x)$ of the function $F(x) = \int\limits_{x^3}^{1} \sqrt{1+t}\,\cos(\pi t)\,dt$, and use it to determine whether $F(x)$ is increasing or decreasing at $x = 1$.

[10] **2.** Find the antiderivative $F(x)$ of the function $f(x)$ that satisfies the given condition:

(a) $f(x) = \dfrac{5^x}{5^x+1}$, $F(0) = 1$. (b) $f(x) = \dfrac{\sec^2 x}{(1+\tan x)^3}$, $F\left(\dfrac{\pi}{4}\right) = 0$.

[16] **3.** Find the following indefinite integrals:

(a) $\displaystyle\int \frac{\ln x}{x^2}\,dx$ (b) $\displaystyle\int \frac{x}{x^2-2x-3}\,dx$ (c) $\displaystyle\int (1-e^x)^2\,dx$.

[11] **4.** Evaluate the following definite integrals (give the exact answers):

(a) $\displaystyle\int_1^e \frac{1}{x\,(1+\ln^2 x)}\,dx$ (b) $\displaystyle\int_0^{\pi/2} x^2 \cos(2x)\,dx$

[8] **5.** Evaluate the given improper integral or show that it diverges:

$$\text{(a)} \quad \int_0^\infty \frac{x}{1+x^2}\,dx \qquad\qquad \text{(b)} \quad \int_0^4 \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$

[15] **6.** **(a)** Sketch the curves $y = x(x^2-2)$ and $y = 2x$, and find the area enclosed by these curves.

(b) Find the volume of a solid obtained by rotating the region bounded by the curve $y = (2 - \sqrt{2x}\,)$ and the lines $y = 0$ and $x = 0$ about the x-axis.

(c) Find the average value of $f(x) = \sin^2 x$ on the interval $[0, \pi]$.

[6] **7.** Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

$$\text{(a)} \quad a_n = \frac{n\,\cos^2(n)}{\sqrt{1+4n^3}} \qquad \text{(b)} \quad a_n = \frac{(2^n+1)^2}{e^n}$$

[15] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

$$\text{(a)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n\, n}{4+n^2} \qquad \text{(b)} \quad \sum_{n=0}^{\infty} (-1)^n e^{-n}\, 2^{n+3} \qquad \text{(c)} \quad \sum_{n=2}^{\infty} \frac{1}{n\,\ln^3(n)}$$

[9] **9.** **(a)** Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n\,3^n}$$

(b) Find the Maclaurin series for $f(x) = x\cos(x^2)$.

(Hint: start with the series for $\cos(t)$, then replace t with x^2.)

[5] **Bonus Question.** It is known that on any given interval $[0, a]$ the average value of some continuous function $f(x)$ is equal to the square of the interval's length, i.e. a^2. Is this information sufficient to find $f(x)$? Find the function f if it is, otherwise explain why it is insufficient.