

CONCORDIA UNIVERSITY
 Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Pages
Final	December 2014	2
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Special Instructions:	Only approved calculators are allowed. Show all your work for full marks.	

[10] 1. (a) Sketch the graph of $f(x) = 2^{-x}$. Using partitioning of the interval $[-2, 2]$ into 4 subintervals of equal length, the definite integral $A = \int_{-2}^2 f(x) dx$ can be approximated by either the leftpoint, or the midpoint, or the rightpoint Riemann sum. Explain which one of these three Riemann sums provides the best approximation for A , and calculate that Riemann sum.

(b) Find the derivative of the function $F(x) = 2e^{-x^2} - \int_{x^2}^1 \sqrt{1+t} e^{-t} dt$, and determine whether $F(x)$ is increasing or decreasing at $x = 1$.

[11] 2. Find the antiderivative $F(x)$ of the function $f(x)$ that satisfies the given condition:

(a) $f(x) = \frac{e^{-3x}}{(e^{-3x} + 1)^3}$, $F(0) = 0$. (b) $f(x) = \tan^2 x$, $F\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

[15] 3. Find the following indefinite integrals:

(a) $\int x \ln(x + 2) dx$ (b) $\int \frac{x}{x^2 - 4x + 3} dx$ (c) $\int x \left(1 + \frac{1}{\sqrt{x}}\right)^2 dx$.

[12] 4. Evaluate the following definite integrals (give the exact values, do **not** approximate):

(a) $\int_0^{\pi/4} \frac{\sec^2(x)}{4 + \tan^2(x)} dx$ (b) $\int_0^{\pi/2} \cos^3(x) \sin^5(x) dx$

[8] 5. Evaluate the given improper integral or show that it diverges:

(a) $\int_{-\infty}^0 x e^{-x^2} dx$ (b) $\int_{-2}^2 \frac{dx}{(x + 2)^{3/2}}$

- [15] 6. (a) Plot the curve $y = \sqrt{4 - x^2}$, and the line $y = 2 - x$, and find the exact value of the area enclosed.
- (b) Find the volume of a solid obtained by rotating the region bounded by the curve $y = \sqrt{3x}$ and the lines $y = 3$ and $x = 0$ about the axis $y = -1$.
- (c) Find the average value of $f(x) = \sin^2(x) \cos^2(x)$ on the interval $[0, \frac{\pi}{2}]$.

- [6] 7. Find the limit of the sequence $\{a_n\}$ at $n \rightarrow \infty$ or prove that it does not exist:

$$(a) \quad a_n = \frac{(-3)^{2n}}{1 + 3n^3 + 9^{n+1}} \quad (b) \quad a_n = \frac{\sqrt{9 + n^2 + 4n^4}}{(n + 3n^{3/2})(4 + \sqrt{n})}$$

- [15] 8. Determine whether the series is divergent or convergent, and if convergent, then is it convergent absolutely or conditionally :

$$(a) \quad \sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{1+n}}{1+n^2} \quad (b) \quad \sum_{n=0}^{\infty} e^{-n} (-3)^{n-1} \quad (c) \quad \sum_{n=2}^{\infty} \frac{\cos(\pi n)}{n \ln(n)}$$

- [8] 9. (a) Find (a) the radius of convergence and (b) the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{n 3^n}$$

- (b) Derive the MacLaurin series of $f(x) = x^3 \ln(1 + 2x^2)$

(HINT: start with the series for $\ln(1 + z)$ where $z = 2x^2$).

- [5] **Bonus Question.** It is known that for some continuous even function, $f(x) = f(-x)$, the average value of $f(x)$ on any given interval $[-a, a]$ ($a > 0$) is equal to the length of the interval. Is this information sufficient to find $f(x)$? Find the function f if it is, otherwise explain why it is insufficient.

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