CVG3120 – Hydrology – Midterm Exam Equation Sheet

Water Budget Equation

$$\Delta S = I - O$$

$$\frac{\Delta S}{\Delta t} = \bar{I} - \bar{O}$$

$$\frac{\partial S}{\partial t} = I(t) - O(t)$$

$$\frac{S_2 - S_1}{t_2 - t_1} = \frac{I_2 + I_1}{2} - \frac{O_2 + O_1}{2}$$

$$\frac{d}{dt}\left(S_s + S_m + S_g + S_i\right) = I_r + I_{sn} - O_{sr} - O_{sb} - O_g - e - e_t$$

$\boldsymbol{S_s}$	surface storage	$\boldsymbol{O_{sr}}$	surface runoff
$\boldsymbol{S_m}$	moisture storage	$\boldsymbol{O_{sb}}$	subsurface flow
$\boldsymbol{S_g}$	groundwater storage	$\boldsymbol{O_g}$	groundwater flow
$\boldsymbol{S_i}$	interception storage	$\boldsymbol{e}$	evaporation
$\boldsymbol{I_r}$	rainfall input	$\boldsymbol{e_t}$	transpiration
$\boldsymbol{I_{sn}}$	snow input		

Watershed Shape Parameters

$L_I = (LL_{ca})^{0.3}$ where all distances are in miles, and L_{ca} is length to the center of area

$F_c = \frac{P}{\sqrt{4\pi A}}$ where P is watershed perimeter and A is watershed area

$R_c = \frac{A}{A_0}$ where A_0 is the area of a circle which has the same perimeter as the watershed

$F_c^2 = \frac{1}{R_c}$

$R_e = \frac{2}{L_m}\sqrt{\frac{A}{\pi}}$ where L_m is maximum length of the watershed parallel to the principal drainage lines

$S = \frac{\Delta E}{L}; S_c = \frac{\Delta E_c}{L_c}; S_{10-8} = \frac{\Delta E_{10-85}}{L_{10-85}}$

Manning's Equation

$V = \frac{1.49}{n} R_h{}^{2/3}\sqrt{S}$ (Imperial Units)

$Q = \frac{1.49}{n} A {R_h}^{2/3} \sqrt{S} = \frac{1.49}{n} \frac{A^{5/3}}{P^{2/3}} \sqrt{S}$ (Imperial Units)

where, V is average stream velocity [ft/s], R_h is hydraulic radius [ft], P is wetted perimeter [ft], S is bed slope [-], n is Manning coefficient, Q is discharge [ft^3/s]

$V = \frac{1}{n} {R_h}^{2/3} \sqrt{S}$ (SI Units)

$Q = \frac{1}{n} A {R_h}^{2/3} \sqrt{S} = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} \sqrt{S}$ (SI Units)

where, V is average stream velocity [m/s], R_h is hydraulic radius [m], P is wetted perimeter [m], S is bed slope [-], n is Manning coefficient, Q is discharge [m^3/s]

$R_h = {}^{A}/_{P}$

$$n = \sum_{k=0}^{k=6} n_k$$

Concentration and Travel Time

$t_t[\text{min}] = \frac{L[\text{m}]}{60[\text{s}] \times V[\text{m/s}]}$; $t_t[\text{min}] = \sum_{i=0}^{i=j} \frac{L_j[\text{m}]}{60[\text{s}] \times V_j[\text{m/s}]}$

$$V = \frac{1.49}{n} {R_h}^{2/3} \sqrt{S} = k\sqrt{S}$$

$d = 0.000131 \frac{nLi}{\sqrt{S}}$ (for wide and shallow flows)

where, L is flow path length [ft], i is in [in/hr] and d is in [ft]

$t_t = \frac{0.938}{i^{0.4}} \left(\frac{nL}{\sqrt{S}}\right)^{0.6}$ (sheet flow travel time)

SCS CN Method

$t_c = 0.00526 L^{0.8} \left(\frac{1000}{\text{CN}} - 9\right)^{0.7} \frac{1}{\sqrt{S}}$ (for homogeneous watersheds of 2000 acres and less (*i.e.* < 8 km^2))

$$\text{CN}_\text{w} = \frac{\sum_{i=1}^{N} A_i \text{CN}_\text{i}}{\sum_{i=1}^{N} A_i}$$

where, A_i is the area corresponding to CN_i

$$V_Q = \frac{(P - I_a)^2}{(P - I_a) + S} = \frac{(P - 0.2S)^2}{P + 0.8S}$$

$$\text{CN} = \frac{1000}{S[\text{in}] + 10}$$

$$\text{CN}_\text{w} = \text{CN}_\text{p}(1 - f) + 98f$$

where, f is the fraction of imperviousness, and CN_P is the curve number for the pervious portion for open space, good condition

$\mathrm{CN_w} = \mathrm{CN_p} + f(98 - \mathrm{CN_p})(1 - 0.5R)$

where, R is the ratio of the unconnected impervious area to the total impervious area

Probability

$$p(x = x_0) = \frac{n}{N}$$

Where x_0 is observed value, n is number of times observed, and N is sample size

$p(x_k) = p(x = x_k) \qquad 0 \le p(x_k) \le 1$

$$\sum_{k=1}^{N} p(x_k) = 1$$

$F(x_k) = p(x \le x_k) = \sum_{j=1}^{N} p(x_j)$

where, $F(x_k)$ is cumulative-mass function

$p(x_1 \le x \le x_2) = \int_{x_1}^{x_2} f(x)dx$

$\int_{-\infty}^{\infty} f(x)dx = 1$

$F(x_k) = p(x \le x_k) = \int_{-\infty}^{x_k} f(x)dx$

$(\bar{x} \text{ or } \mu) = \int_{-\infty}^{\infty} xf(x)dx$ (continuous random variables)

$(\bar{x} \text{ or } \mu) = \sum_{-\infty}^{\infty} x_i p(x_i)$; $(\bar{x} \text{ or } \mu) = \frac{1}{n}\sum_{i=1}^{n} x_i$ (discrete random variables)

$(S^2 \text{ or } \sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ (continuous random variables)

$(S^2 \text{ or } \sigma^2) = \sum_{i=1}^{n} (x_i-\mu)^2 p(x_i)$; $(S^2 \text{ or } \sigma^2) = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1} = \frac{\left[\sum_{i=1}^{n} x_i{}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right]}{n-1}$ (discrete random variables)

$(g \text{ or } \gamma) = \int_{-\infty}^{\infty} (x-\mu)^3 f(x)dx$ (continuous random variables)

$(g \text{ or } \gamma) = \sum_{i=1}^{n} (x_i-\mu)^3 p(x_i)$; $(g \text{ or } \gamma) = \frac{1}{n}\sum_{i=1}^{n} (x_i-\mu)^3 = \frac{n\sum_{i=1}^{n}(x_i-\mu)^3}{(n-1)(n-2)S^3}$ (discrete random variables)

Normal (Gaussian) Distribution

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ for $-\infty < z < +\infty$

$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}z^2\right]$ for $-\infty < z < +\infty$; $z = \frac{x-\mu}{\sigma}$

Binomial Distribution

${C_n}^k = \frac{n!}{k!(n-k)!}$; $p\{k|n\} = {C_n}^k p^k (1-p)^{n-k}$ (k events occurring in n trials)

Recurrence Interval or Return Period

$$p_{Q_{max}} = p(Q > Q_{max}) = \int_{Q_{max}}^{\infty} f(x)\, dx = 1 - F(Q_{max})$$

$$T_{Q_{max}} = \frac{1}{p_{Q_{max}}} = \frac{1}{1 - F(Q_{max})}$$

$$\text{Risk} = 1 - \text{probability of no occurrence} = 1 - \text{p}(0) = 1 - \left(1 - p_{Q_{max}}\right)^n$$

$$\text{Reliability} = 1 - \text{Risk}$$

Population Models

Distribution	Parameters	PDF
Exponential	a	$f(x;a) = \begin{cases} ae^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases}$
Normal	$\mu,\ \sigma$	$f(x;\mu,\ \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$
Lognormal	$\mu,\ \sigma$	$f(x;\mu,\ \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right]$
Gamma	$a,\ b$	$f(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{\frac{-x}{b}}$
Pearson Type III	$a,\ b,\ c$	$f(x) = \frac{1}{a\Gamma(b)}\left(\frac{x-c}{a}\right)^{b-1} \exp\left(-\frac{x-c}{a}\right)$
Log-Pearson Type III	$a,\ b,\ c$	$f(x) = \frac{1}{ax\Gamma(b)}\left(\frac{\ln x-c}{a}\right)^{b-1} \exp\left(-\frac{\ln x-c}{a}\right)$

$\mu = \int_{-\infty}^{+\infty} x f(x;a)dx = \frac{1}{a}$ (exponential distribution)

Plotting Position Formulas

$p_i = \frac{i}{n+1}$ (Weibull); $p_i = \frac{i-0.5}{n}$ (Hazen); $p_i = \frac{i-0.4}{n+0.2}$ (Cunnane)

Frequency Factors

$Q_T = \bar{X} + K(T,\ \text{Distribution})S$ (normal distribution) where, $\bar{X}$ is generally the mean of the data set and S its standard deviation

$Q_T = e^{\bar{X}+K(T,\text{ Distribution})S}$ (Lognormal and Log-Pearson distribution)

Single Return Period Equations

$Q_T \ = \ b_0 X_1{}^{b_1} X_2{}^{b_2} \ldots X_p{}^{b_p}$

where, Q_T is peak discharge, X_j is the j^{th} *watershed* characteristic and b_j is a regression coefficient (j = 1, 2, … p)

Areal Precipitation

$\bar{P} = \sum_{i=1}^{N} P_i \left(\frac{A_i}{A_W}\right)$ (Arithmetic Method)
$\bar{P} = \sum_{i=1}^{N} P_i \left(\frac{A_i}{A_T}\right)$ (Thiessen Polygon Method)
$\bar{P} = \sum_{i=1}^{N} \frac{P_i}{N}$ (Isohyetal Method)

Missing Precipitation Record

$a_i = \frac{\mathbf{1}}{\boldsymbol{m}} \Rightarrow \ P_x = \sum_{i=1}^{m} P_i(a_i) = \sum_{i=1}^{m} \left(\frac{P_i}{m}\right)$ (Arithmetic Average Method)
$a_i = \frac{1/D_i^2}{\sum_{i=1}^{m} 1/D_i^2} \Rightarrow \ P_x = \sum_{i=1}^{m} P_i(a_i)$ (Inverse Distance Method)
$a_i = \frac{N_x}{m \times N_i} \Rightarrow \ P_x = \sum_{i=1}^{m} P_i(a_i)$ (Normal ratio method)
where, m is the number of stations, D_i is the distance between station, N_x is the normal precipitation at the target station, N_i is the normal precipitation