CVG3120 – Hydrology – Assignment # 3 (Solutions)

Problem 1

$V_{30} = 30\ mi/hr$ at 30 feet above the water surface

Meyer mass transfer equation:

$$E = b_0\,(1 + 0.1\,V_{30})(e_s - e) \quad (1)$$

- Find the best value of
- Assess the accuracy of the fitted equation

Solution

Estimate e_s using the following equation [Table 1- column (b)]:

$$e_s = 33.8639\,[(7.38\ \times 10^{-3}T + 0.8072)^8 - 1.9 \times 10^{-5}|1.8T + 48| + 13.16 \times 10^{-4}] \quad (2)$$

where, e_s is saturation pressure (mb) and T is temperature (˚C) [Table 1- column (a)].

Estimate e using the following equation [Table 1- column (c)]:

$$e = R_h\ \times\ e_s \quad (3)$$

where, R_h is relative humidity.

Replace the calculated values for e_s and e in eq. (1) and calculate b_0 [Table 1- column (d)]. Use the average of b_0 values to recalculate E in eq. (1) [Table 1- column (e)]. Compare the two values for E by calculating the true relative errors [eq. (4)] and assess the accuracy of the fitted equation [Table 1- column (f)]:

$$Error\ (\%) = \frac{|E - E_{approx.}|}{E} \times 100 \quad (4)$$

Table 1

	(a)	(b)		(c)			(d)	(e)	(f)
T (F)	**T (C)**	**e_s (mb)**	**R_h**	**e (mb)**	**V_{30} (mph)**	**E (in/d)**	**b_0**	**E (in/d)**	**Difference (%)**
63	17.2	19.7	0.74	14.5	7.5	0.08	0.00895	0.08621	7.77
71	21.7	25.9	0.66	17.1	9.2	0.14	0.00828	0.16299	16.42
76	24.4	30.6	0.63	19.3	8.4	0.18	0.00863	0.20104	11.69
68	20.0	23.4	0.68	15.9	6.3	0.13	0.01066	0.11756	9.57
62	16.7	19.0	0.76	14.4	6.8	0.09	0.01176	0.07376	18.05
54	12.2	14.2	0.77	11.0	9.1	0.06	0.00959	0.06032	0.53
						Avg.	**0.00964**		**10.67**

Based on the average value for true relative error (10.67%), the fitted equation does not satisfy enough accuracy for further analysis.

Problem 2

Measured atmospheric pressure: $P_a = 1013\ Mb$

Temperature of the wet bulb: $T_w = 12℃$

Room temperature: $T_a = 20℃$

Find the followings:

- Absolute humidity (ρ_v)
- Relative humidity (R_h)
- Actual vapor pressure (e)
- Vapor pressure deficit (e - e_s)
- The dew point temperature (T_d)

Solution

- **Actual vapor pressure (e):**

Find saturation vapor pressure for $T_w = 12℃$ from Table 14.1 (text book):

$$e_s = 10.52\ mm\ Hg$$

$$1\ mm\ Hg \quad 1.36\ mb$$

$$e_s = 10.52\ \times 1.36 = 14.31\ mb$$

Use eq. (5) to calculate actual vapor pressure:

$$e = e_s - \frac{P_a(C_p)}{0.622 H_v}(T_a - T_w) \qquad (5)$$

where,

$C_p = 0.2396\ (\frac{cal}{g \times ℃})$

$$H_V = 597.3 - 0.564\ \times T_w \left(\frac{cal}{g}\right) = 597.3 - 0.564\ \times 12\ \left(\frac{cal}{g}\right) = 590.532\ \left(\frac{cal}{g}\right)$$

Substitude the values in eq. (1):

$$e = 14.31\ Mb - \frac{1013Mb\ \times 0.2396}{0.622\ \times\ 590.532}(20 - 12)$$

$$\boldsymbol{e = 8.740\ mb}$$

- **Relative humidity (R_h):**

Use eq. (3) to calculate actual vapor pressure:

$$\boldsymbol{R_h} = \frac{e}{e_s} = \frac{8.740}{14.31} = \mathbf{0.611}$$

- **Absolute humidity (ρ_v):**

Use eq. (6) to calculate actual vapor pressure:

$$\rho_v = 0.622\ \frac{e}{RT} \qquad (6)$$

where, $R = 2.8704\ \times\ 10^{-3}$, and T is absolute temperature (° K)

$$\boldsymbol{\rho_v} = 0.622\ \frac{8.740}{2.8704 \times\ 10^{-3} \times (273 + 20)} = \mathbf{6.464}\ (\frac{\boldsymbol{mg}}{\boldsymbol{cm^3}})$$

- **Vapor pressure deficit (e - e_s):**

$e = 8.740$ and $e_s = 14.31$, therefore:

$$e_s - e = 14.31 - 8.740 = \mathbf{5.570}$$

- **The dew point temperature (T_d):**

$$T_a - T_d = (14.55 + 0.114\,T_a)(1 - R_h) + [(2.5 + 0.007\,T_a)(1 - R_h)]^3 + (15.9 + 0.117\,T_a)(1 - R_h)^{14}$$

$$= (14.55 + 0.114 \times 20) \times (1 - 0.611) + [(2.5 + 0.007 \times 20) \times (1 - 0.611)]^3 + (15.9 + 0.117 \times 20) \times (1 - 0.611)^{14}$$

$$T_a - T_d = 7.63℃$$

$$T_d = 20℃ - 7.63℃$$

$$\mathbf{T_d = 12.37}\ ℃$$

Problem 3

For the following watershed (Figure 1), compare the average depth of storm rainfall using the Thiessen Method:

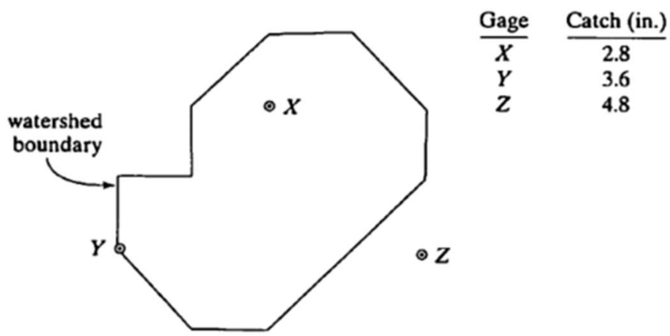

Figure 1

Solution

Connect gages with lines and form a triangle. Find the midpoints of each side of the triangle and draw the perpendicular lines (Figure 2) and estimate the area attributed to each gage. Then calculate the average depth of storm rainfall. Results are shown in Table 2:

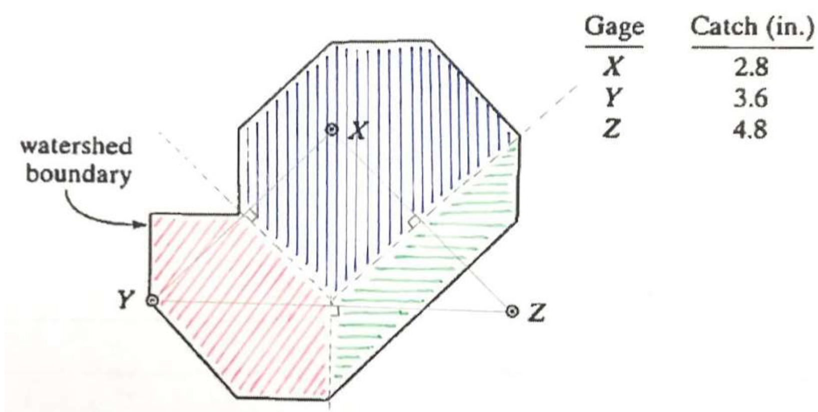

Figure 2

Table 2

	P_i (in)	Area	$P_i \times$ Area
X	2.8	11.37	31.84
Y	3.6	6.15	22.14
Z	4.8	3.92	18.82
	Sum	21.44	72.8
		Average	**3.40 in**

Problem 4

The long-term average annual rainfall (in) and sorm-event total (in) for six gages are given in Table 3:

1) Use three different methods to compute the rainfall at gags **5** and **6**.
2) Using Thiessen method, estimate the average precipitation on the watershed with the given coordinates with the data from **a)** stations **1 to 4**, **b)** stations **1 to 6**.

Table 3

Station	1	2	3	4	5	6	Σ
X	-100	200	100	-75	70	-55	
Y	50	-75	100	25	30	80	
Annual (N_i), in.	29.1	34.4	30.9	30.2	32	25	
Storm (P_i), in.	1.3	2.7	1.8	1.9	**? = 1.870**	**? = 1.628**	
Distance from Station 5 (D_5)	171.17	167.11	76.16	145.09	0	0	
$1/(D_5)^2$	0.000034	0.000036	0.000172	0.000048	0	0	0.000290
a_5	0.117241	0.124138	0.593103	0.165517			
$\mathbf{P_5}$	0.152	0.335	1.068	0.314			**1.870**
Distance from Station 6 (D_6)	54.08	298.41	156.28	58.25	134.63	0	
$1/(D_6)^2$	0.000342	0.000011	0.000041	0.000293	0.000055	0	0.000742
a_6	0.460916	0.014825	0.055256	0.394879	0.074124		
$\mathbf{P_6}$	0.599	0.040	0.099	0.750	0.139		**1.628**

Solution 1)

- Arithmetic Average Method:

$$P_x = \sum_{i=1}^{m} \frac{P_i}{m}$$

where, number of gages (m) is 4. Therefore:

$$\boldsymbol{P_5} = \boldsymbol{P_6} = \sum_{i=1}^{4} \frac{P_i}{4} = \frac{1}{4}[1.3 + 2.7 + 1.8 + 1.9] = \mathbf{1.925}\ \boldsymbol{in}$$

- The Normal Ratio Method:

$$P_x = \sum_{i=1}^{m} P_i \times \left(\frac{N_x}{m \times N_i}\right)$$

$$\boldsymbol{P_5} = \frac{32}{4} \times \left(\frac{1.3}{29.1} + \frac{2.7}{34.4} + \frac{1.8}{30.9} + \frac{1.9}{30.2}\right) = \mathbf{1.955}$$

$$P_6 = \frac{25}{4} \times \left(\frac{1.3}{29.1} + \frac{2.7}{34.4} + \frac{1.8}{30.9} + \frac{1.9}{30.2}\right) = \mathbf{1.527}$$

- The Inverse Distance Method:

$$P_x = \sum_{i=1}^{m} P_i \times a_i \quad \text{and} \quad a_i = \left(\frac{\frac{1}{D_i^2}}{\sum_{i=1}^{m} \frac{1}{D_i^2}}\right)$$

For gage 5:

$$P_5 = \frac{1.3 \times \left(\frac{1}{171.17^2}\right)}{0.000290} + \frac{2.7 \times \left(\frac{1}{167.11^2}\right)}{0.000290} + \frac{1.8 \times \left(\frac{1}{76.16^2}\right)}{0.000290} + \frac{1.9 \times \left(\frac{1}{145.09^2}\right)}{0.000290}$$

$$P_5 = 0.152 + 0.335 + 1.068 + 0.314$$

$$\boldsymbol{P_5 = 1.870 \quad in}$$

Similar calculations for gage 6 will result in:

$$\boldsymbol{P_6 = 1.628 \ in}$$

The results are shown in Table 3 and are compared in Table 4:

Table 4

	Arithmetic Average	Normal Ratio	Inverse Distance
P_5 (in)	1.925	1.955	1.870
P_6 (in)	1.925	1.527	1.628

As shown in the table, the Arithmetic Average Method, results in the same value for both gages 5 and 6 and considers only the average rainfall of other gages, which indicates that the extrapolation was not accurate enough. While the two other methods, also consider the distance between the gages, and therefore lead in more reliable results.

<u>Solution 2)</u>

Follow the steps for Problem 3 and after estimating the area attributed to each gage, approximate the average precipitation on the watershed. Results are shown in Figures 3 and Table 5 (for part a of the problem) and Figure 4 and Table 6 (for part b of the problem):

Figure 3 (Stations 1 to 4)

Table 5

Gage	P_i (in)	Area	$P_i \times$ Area
1	29.1	0.00	0.00
2	34.4	80.00	2752.00
3	30.9	2088.89	64546.67
4	30.2	11344.44	353946.67
	Sum	13513.33	421245.33
		Average	**31.17 in**

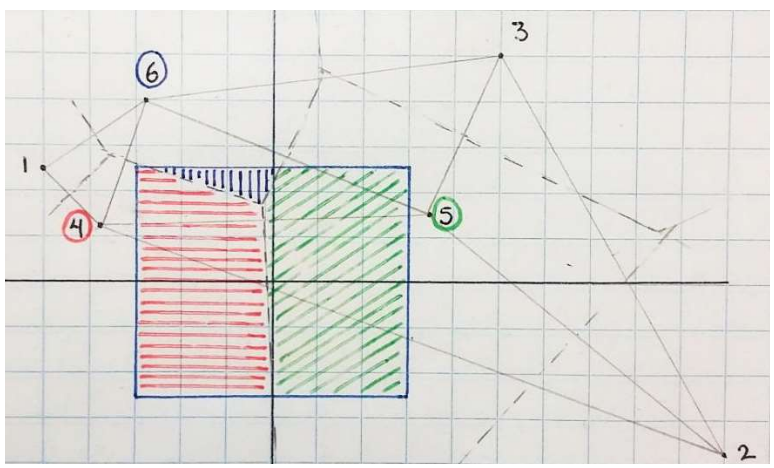

Figure 4 (Stations 1 to 6)

Table 6

Gage	P_i (in)	Area	$P_i \times$ Area
1	29.1	0.00	0.0
2	34.4	0.00	0.0
3	30.9	0.00	0.0
4	30.2	6311.11	190595.5
5	32	6755.56	216177.9
6	25	444.44	11111.0
	Sum	13511.11	417884.4
		Average	**30.93 in**