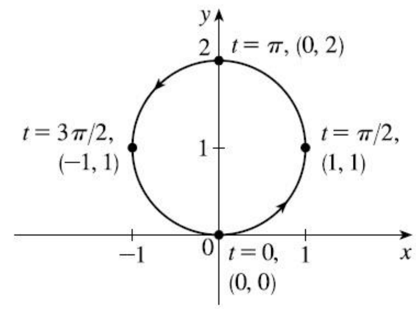
Solutions for Assignment 1, MAST 218

10.1

8. $x = \sin t, \quad y = 1 - \cos t, \quad 0 \le t \le 2\pi$

(a)

t	0	$\pi/2$	π	$3\pi/2$	2π
x	0	1	0	-1	0
y	0	1	2	1	0



(b) $x = \sin t$, $y = 1 - \cos t$ $\quad$ [or $y - 1 = -\cos t$] $\quad \Rightarrow$

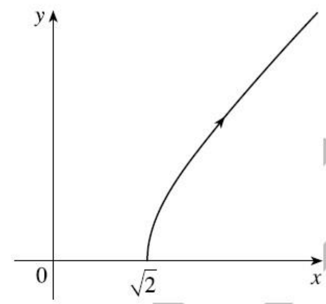
$$x^2 + (y-1)^2 = (\sin t)^2 + (-\cos t)^2 \quad \Rightarrow \quad x^2 + (y-1)^2 = 1.$$

As t varies from 0 to 2π, the circle with center $(0, 1)$ and radius 1 is traced out.

16. (a) $x = \sqrt{t+1} \quad \Rightarrow \quad x^2 = t + 1 \quad \Rightarrow \quad t = x^2 - 1.$

$y = \sqrt{t-1} = \sqrt{(x^2-1)-1} = \sqrt{x^2-2}$. The curve is the part of the hyperbola $x^2 - y^2 = 2$ with $x \ge \sqrt{2}$ and $y \ge 0$.

(b)



24. (a) From the first graph, we have $1 \le x \le 2$. From the second graph, we have $-1 \le y \le 1$. The only choice that satisfies either of those conditions is III.

(b) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

(c) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph, we have $0 \le y \le 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y do the same thing. Choice II satisfies these conditions.

10.2

6. $x = e^t \sin \pi t,\ y = e^{2t};\quad t = 0.\ \dfrac{dy}{dt} = 2e^{2t},\ \dfrac{dx}{dt} = e^t(\pi \cos \pi t) + (\sin \pi t)e^t = e^t(\pi \cos \pi t + \sin \pi t)$, and

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2e^{2t}}{e^t(\pi \cos \pi t + \sin \pi t)} = \dfrac{2e^t}{\pi \cos \pi t + \sin \pi t}$. When $t = 0$, $(x, y) = (0, 1)$ and $dy/dx = 2/\pi$, so an equation of the tangent to the curve at the point corresponding to $t = 0$ is $y - 1 = \frac{2}{\pi}(x - 0)$, or $y = \frac{2}{\pi}x + 1$.

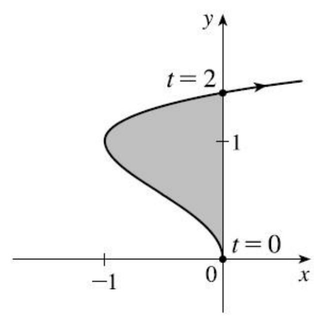
16. $x = \cos t,\ y = \sin 2t,\ 0 < t < \pi \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2\cos 2t}{-\sin t} \quad \Rightarrow$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{\dfrac{(-\sin t)(-4\sin 2t) - (2\cos 2t)(-\cos t)}{(-\sin t)^2}}{-\sin t} = \frac{(\sin t)(8\sin t\ \cos t) + [2(1 - 2\sin^2 t)](\cos t)}{(-\sin t)\sin^2 t}$$

$$= \frac{(\cos t)(8\sin^2 t + 2 - 4\sin^2 t)}{(-\sin t)\sin^2 t} = -\frac{\cos t}{\sin t} \cdot \frac{4\sin^2 t + 2}{\sin^2 t} \qquad [\,(-\cot t) \cdot \text{positive expression}]$$

The curve is CU when $\dfrac{d^2y}{dx^2} > 0$, that is, when $-\cot t > 0 \quad \Leftrightarrow \quad \cot t < 0 \quad \Leftrightarrow \quad \frac{\pi}{2} < t < \pi$.

32. The curve $x = t^2 - 2t = t(t - 2)$, $y = \sqrt{t}$ intersects the y-axis when $x = 0$, that is, when $t = 0$ and $t = 2$. The corresponding values of y are 0 and $\sqrt{2}$. The shaded area is given by



$$\int_{y=0}^{y=\sqrt{2}} (x_R - x_L)\,dy = \int_{t=0}^{t=2} [0 - x(t)]\,y'(t)\,dt = -\int_0^2 (t^2 - 2t)\left(\frac{1}{2\sqrt{t}}\,dt\right)$$

$$= -\int_0^2 \left(\tfrac{1}{2}t^{3/2} - t^{1/2}\right)dt = -\left[\tfrac{1}{5}t^{5/2} - \tfrac{2}{3}t^{3/2}\right]_0^2$$

$$= -\left(\tfrac{1}{5} \cdot 2^{5/2} - \tfrac{2}{3} \cdot 2^{3/2}\right) = -2^{1/2}\left(\tfrac{4}{5} - \tfrac{4}{3}\right)$$

$$= -\sqrt{2}\left(-\tfrac{8}{15}\right) = \tfrac{8}{15}\sqrt{2}$$

42. $x = e^t - t,\ y = 4e^{t/2},\ 0 \le t \le 2.\quad dx/dt = e^t - 1$ and $dy/dt = 2e^{t/2}$, so

$(dx/dt)^2 + (dy/dt)^2 = (e^t - 1)^2 + (2e^{t/2})^2 = e^{2t} - 2e^t + 1 + 4e^t = e^{2t} + 2e^t + 1 = (e^t + 1)^2$. Thus,

$$L = \int_0^2 \sqrt{(e^t + 1)^2}\,dt = \int_0^2 \left|e^t + 1\right|\,dt = \int_0^2 (e^t + 1)\,dt = \left[e^t + t\right]_0^2 = (e^2 + 2) - (1 + 0) = e^2 + 1.$$

10.3

20. $r^2 \sin 2\theta = 1 \quad \Leftrightarrow \quad r^2(2 \sin\theta \, \cos\theta) = 1 \quad \Leftrightarrow \quad 2(r\cos\theta)(r\sin\theta) = 1 \quad \Leftrightarrow \quad 2xy = 1 \quad \Leftrightarrow \quad xy = \frac{1}{2}$, a hyperbola centered at the origin with foci on the line $y = x$.

26. $x^2 - y^2 = 4 \quad \Leftrightarrow \quad (r\cos\theta)^2 - (r\sin\theta)^2 = 4 \quad \Leftrightarrow \quad r^2\cos^2\theta - r^2\sin^2\theta = 4 \quad \Leftrightarrow \quad r^2(\cos^2\theta - \sin^2\theta) = 4 \quad \Leftrightarrow$ $r^2 \cos 2\theta = 4$

32. $r = 1 + 2\cos\theta$