Examples

1. Find the domains of following functions.
   a) $f(x) = \sqrt{x-3}$ b) $f(x) = \frac{1}{x^2+6x+5}$

   Solutions:

   a) Recall for the functions contain an even root, the inside part need to be larger or equal to zero. Thus, $x - 3 \geq 0 \Rightarrow x \geq 3$, the domain is $[3, +\infty)$. Also, note that the range is $[0, +\infty)$, since the square root of any nonnegative number is nonnegative.
   b) Recall for the rational functions, the denominator cannot be equal to zero.

   Thus, $x^2 + 6x + 5 \neq 0 \Rightarrow (x+1)(x+5) \neq 0 \Rightarrow x \neq -1 \; and \; x \neq$-5.

   Then, the domain is $(-\infty, -5) \cup (-5, -1) \cup (-1, +\infty)$.

2. An open rectangular box with volume 2 has a square base. Express the surface area of the box as a function of the length of a side of the base.

   Solutions:

   Let x denotes the length of a side of the base and h denotes the height.

   We know the volume is 2, so we would get the equation that $= h * x^2 = 2 \Rightarrow h = \frac{2}{x^2}$.
   Also, the surface area can be expressed into $S(x) = x^2 + 4 * h * x$. Since this expression includes two variables h and x, we need to replace the h by $\frac{2}{x^2}$.
   Hence, the final expression is $S(x) = x^2 + \frac{8}{x}, x > 0$.

3. Given that two points (-2,-8) and (3,7) on the line $y = mx + b$. Find the value of m and b.

   Solutions: Recall that the $slope = \frac{\Delta y}{\Delta x}$.

   Thus, $m = \frac{-8-7}{-2-3} = 3, then \; y = 3x + b.$

   Plugging any of these two points into the equation, we could solve for b. So,

   $-8 = 3 * (-2) + b \Rightarrow b = -2$.

   Also, we could plug both of them into the function, then we will have two equations. Solving the system of equations, we cloud get the value of m and b.

4. Given that the point (1,3) on the line l which is parallel to the line $y = 4x + 3$. Find the expression of the line l. If the line l' which is perpendicular to the line $y = 4x + 3$. Find the expression of the line l'.

   Solutions:

   Recall that if two lines parallel to each other, then $m_1 = m_2$. If two lines perpendicular to each other, then $m_1 m_2 = -1$.

   Hence, for line l, $y = 4x + b$. Plug the point into the equation, then $3 = 4 * 1 + b \Rightarrow b = -1$.

   So, for line l, $y = 4x - 1$.

   Hence, for line l', $y = -\frac{x}{4} + b$. Plug the point into the equation, then $3 = -\frac{1}{4} + b \Rightarrow b = \frac{13}{4}$.

   So, for line l', $y = -\frac{x}{4} + \frac{13}{4}$.