(A)

MAT 1320 B Fall 2016 October 5th, 11:30 Prof. Desjardins

TEST #1

Max = 15

Name: Solutions

Student Number: ________________

- Time: 80 min.
- No calculators are permitted.
- There are 5 multiple choice questions worth 1 mark each and 3 problems worth 10 marks.
- For the multiple choice questions, circle the letter of your choice.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.
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Signature: ________________

(A)

1. [1 point] What is the domain of the function $f(x) = \sqrt{4 - x^2}$?

(A) $x \geq 2$ (B) $-2 \leq x \leq 2$ (C) $x \leq -2$

(D) $x < 2$ (E) all x (F) $x \leq -2$ or $x \geq 2$

need $4 - x^2 \geq 0$

$4 \geq x^2$

ie $x^2 \leq 4$

2. [1 point] Find a formula for the inverse of $f(x) = \ln(x + 6)$.

(A) e^{x+6} (B) $e^x + 6$ (C) $e^x - 6$

(D) $x + 6$ (E) e^{x-6} (F) $6 - e^x$

$y = \ln(x+6)$

So $x = \ln(y+6)$

$e^x = y + 6$

3. [1 point] What is the equation of the tangent line to the curve $y = f(x) = 2x + 3x^2$ at the point $(1, 5)$?

(A) $y = 32x - 27$ (B) $y = 6x - 1$ (C) $y = 8x - 3$

(D) $y = 3x + 2$ (E) $y = 2x + 3$ (F) $y = 2x + 3x^2$

$f'(x) = 2 + 6x$

$f'(1) = 8$

$y - 5 = 8(x - 1)$

A

4. [1 point] If a function $f(x)$ is continuous at $x = a$, then it must be differentiable there.

(A) TRUE (B) FALSE

5. [1 point] What is $\lim_{x\to\infty} \dfrac{5x^3 + 7x^2 - 8x + 9}{11x^2 + 7x - 6}$?

(A) 3/2 (B) 0 (C) ∞ (D) $-\infty$ (E) 5/11 (F) 11/5

$$= \lim_{x\to\infty} \frac{5x + 7 - 8/x + 9/x^2}{11 + 7/x - 6/x^2}$$

(A)

6. [2 points] Find the limit $\lim_{x\to 2}\dfrac{\sqrt{x^2+12}-4}{x-2}$. You must do this properly, showing all steps.

$$\lim_{x\to 2}\frac{\sqrt{x^2+12}-4}{x-2} = \lim_{x\to 2}\left(\frac{\sqrt{x^2+12}-4}{x-2}\right)\left(\frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}\right)$$

$$= \lim_{x\to 2}\frac{((x^2+12)-16)}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \lim_{x\to 2}\frac{x^2-4}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \lim_{x\to 2}\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x^2+12}+4)} = \lim_{x\to 2}\frac{x+2}{\sqrt{x^2+12}+4} = \frac{4}{4+4} = \boxed{\frac{1}{2}}$$

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x)=\dfrac{2x^2}{x+3}$. Then verify your answer with the Quotient Rule.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{2(x+h)^2}{x+h+3}-\frac{2x^2}{x+3}}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{2(x+3)(x^2+2xh+h^2)-2x^2(x+h+3)}{(x+3)(x+h+3)}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{\cancel{2x^3}+\cancel{6x^2}+4x^2h+12xh+2xh^2+6h^2-\cancel{2x^3}-2x^2h-\cancel{6x^2}}{(x+3)(x+h+3)}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{2x^2h+12xh+2xh^2+6h^2}{(x+3)(x+h+3)}\right)$$

$$= \lim_{h\to 0}\frac{2x^2+12x+2xh+6h}{(x+3)(x+h+3)} = \boxed{\frac{2x^2+12x}{(x+3)^2}}$$

QR: $$f'(x) = \frac{4x(x+3)-2x^2(1)}{(x+3)^2} = \boxed{\frac{2x^2+12x}{(x+3)^2}}$$

A

8. [5 points] Find the first derivatives of the following functions.

(a) $f(x) = e^{-3x}\sin(4x^2)$

$$\boxed{f'(x) = -3e^{-3x}\sin(4x^2) + 8xe^{-3x}\cos(4x^2)}$$

(b) $g(t) = \sqrt{3t^2 + 7t - 2} = (3t^2 + 7t - 2)^{1/2}$

$$\boxed{g'(t) = \frac{1}{2}(3t^2 + 7t - 2)^{-1/2}(6t + 7)}$$

$$= \boxed{\frac{6t+7}{2\sqrt{3t^2+7t-2}}}$$

(c) $\varphi(\theta) = \tan^2(3e^\theta)$

$$\boxed{\varphi'(\theta) = 2\tan(3e^\theta)\sec^2(3e^\theta)(3e^\theta)}$$

$$= \boxed{6e^\theta\tan(3e^\theta)\sec^2(3e^\theta)}$$

(d) $p(t) = 5^{\sqrt{t}}$

$$\boxed{p'(t) = 5^{\sqrt{t}}\ln 5\left(\frac{1}{2}t^{-1/2}\right)}$$

$$= \boxed{\frac{1}{2\sqrt{t}}\ln 5\ 5^{\sqrt{t}}}$$

(e) $y = e^{x\cos(2x)}$

$$\boxed{\frac{dy}{dx} = e^{x\cos(2x)}\left(\cos(2x) - 2x\sin(2x)\right)}$$

(B)

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(B)

1. [1 point] What is the domain of the function $f(x) = \sqrt{x^2 - 4}$?

(A) $x \geq 2$ (B) $-2 \leq x \leq 2$ (C) $x \leq -2$

(D) $x < 2$ (E) all x (F) $x \leq -2$ or $x \geq 2$

need $x^2 - 4 \geq 0$

$x^2 \geq 4$

2. [1 point] Find a formula for the inverse of $f(x) = \ln(x - 6)$.

(A) e^{x+6} (B) $e^x + 6$ (C) $e^x - 6$

(D) $x + 6$ (E) e^{x-6} (F) $6 - e^x$

$y = \ln(x-6)$

$x = \ln(y-6)$

$e^x = y - 6$

3. [1 point] What is the equation of the tangent line to the curve $y = f(x) = 3x + 2x^2$ at the point $(1, 5)$?

(A) $y = 32x - 27$ (B) $y = 6x - 1$ (C) $y = 8x - 3$

(D) $y = 3x + 2$ (E) $y = 7x - 2$ (F) $y = 3x + 2x^2$

$f'(x) = 3 + 4x$

$f'(1) = 7$

$y - 5 = 7(x - 1)$

B

4. [1 point] If a function $f(x)$ is differentiable at $x = a$, then it must be continuous there.

(A) TRUE (B) FALSE

5. [1 point] What is $\lim_{x\to\infty} \dfrac{5x^3 + 7x^2 - 8x + 9}{11x^3 + 7x^2 - 6x - 13}$?

(A) 3/2 (B) 0 (C) ∞ (D) $-\infty$ (E) 5/11 (F) 11/5

$$= \lim_{x\to\infty} \frac{5 + 7/x - 8/x^2 + 9/x^3}{11 + 7/x - 6/x^2 - 13/x^3}$$

(B)

6. [2 points] Find the limit $\lim_{x\to 1}\dfrac{\sqrt{x^2+8}-3}{x-1}$. You must do this properly, showing all steps.

$$\lim_{x\to 1}\frac{\sqrt{x^2+8}-3}{x-1} = \lim_{x\to 1}\left(\frac{\sqrt{x^2+8}-3}{x-1}\right)\left(\frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}\right)$$

$$= \lim_{x\to 1}\frac{(x^2+8)-9}{(x-1)(\sqrt{x^2+8}+3)}$$

$$= \lim_{x\to 1}\frac{x^2-1}{(x-1)(\sqrt{x^2+8}+3)}$$

$$= \lim_{x\to 1}\frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x^2+8}+3)} = \lim_{x\to 1}\frac{x+1}{\sqrt{x^2+8}+3} = \frac{2}{3+3} = \boxed{\frac{1}{3}}$$

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x)=\dfrac{3x^2}{x+4}$. Then verify your answer with the Quotient Rule.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{1}{h}\left[\frac{3(x+h)^2}{x+h+4} - \frac{3x^2}{x+4}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{3(x+4)(x^2+2xh+h^2)-3x^2(x+h+4)}{(x+4)(x+h+4)}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{\cancel{3x^3}+\cancel{12x^2}+6x^2h+24xh+3xh^2+12h^2-\cancel{3x^3}-3x^2h-\cancel{12x^2}}{(x+4)(x+h+4)}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{3x^2h+24xh+3xh^2+12h^2}{(x+4)(x+h+4)}\right)$$

$$= \lim_{h\to 0}\frac{3x^2+24x+3xh+12h}{(x+4)(x+h+4)} = \boxed{\frac{3x^2+24x}{(x+4)^2}}$$

QR: $$f'(x) = \frac{6x(x+4)-3x^2(1)}{(x+4)^2} = \boxed{\frac{3x^2+24x}{(x+4)^2}}$$

(B)

8. [5 points] Find the first derivatives of the following functions.

(a) $f(x) = e^{-4x}\cos(x^2)$

$\boxed{f'(x) = -4e^{-4x}\cos(x^2) - 2xe^{-4x}\sin(x^2)}$

(b) $g(t) = \sqrt{4t^2 + 6t - 3}$ $= (4t^2+6t-3)^{1/2}$

$\boxed{g'(t) = \frac{1}{2}(4t^2+6t-3)^{-1/2}(8t+6)} = \boxed{\dfrac{4t+3}{\sqrt{4t^2+6t-3}}}$

(c) $\varphi(\theta) = \sec^2(2e^\theta)$

$\boxed{\varphi'(\theta) = 2\sec(2e^\theta)\sec(2e^\theta)\tan(2e^\theta)(2e^\theta)}$

$= \boxed{4e^\theta \sec^2(2e^\theta)\tan(2e^\theta)}$

(d) $p(t) = 6^{\sqrt{t}}$

$\boxed{p'(t) = 6^{\sqrt{t}}\ln 6\left(\frac{1}{2}t^{-1/2}\right)} = \boxed{\frac{1}{2\sqrt{t}}\ln 6\ 6^{\sqrt{t}}}$

(e) $y = e^{x\sin(3x)}$

$\boxed{\frac{dy}{dx} = e^{x\sin(3x)}(\sin(3x) + 3x\cos(3x))}$

C

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C

1. [1 point] What is the domain of the function $f(x) = \sqrt{16 - x^2}$?

(A) $x \geq 4$ (B) all x (C) $x \leq -4$

(D) $x < 4$ (E) $-4 \leq x \leq 4$ (F) $x \leq -4$ or $x \geq 4$

need $16 - x^2 \geq 0$

$16 \geq x^2$

$x^2 \leq 16$

2. [1 point] Find a formula for the inverse of $f(x) = \ln(x + 4)$.

(A) $e^x - 4$ (B) $e^x + 4$ (C) e^{x+4}

(D) $x + 4$ (E) e^{x-4} (F) $4 - e^x$

$y = \ln(x+4)$

$x = \ln(y+4)$

$e^x = y+4$

3. [1 point] What is the equation of the tangent line to the curve $y = f(x) = x + 4x^2$ at the point $(1, 5)$?

(A) $y = 9x - 4$ (B) $y = 6x - 1$ (C) $y = 8x - 3$

(D) $y = 3x + 2$ (E) $y = 2x + 3$ (F) $y = x + 4x^2$

$f'(x) = 1 + 8x$

$f'(1) = 9$

$y - 5 = 9(x-1)$

C

4. [1 point] If a function $f(x)$ is continuous at $x = a$, then it must be differentiable there.

(A) FALSE (B) TRUE

5. [1 point] What is $\lim_{x\to\infty} \dfrac{5x^3 + 7x^2 - 8x + 9}{11x^4 + 7x^3 - 6x^2 + 17x - 5}$?

(A) 3/4 (B) 0 (C) ∞ (D) $-\infty$ (E) 5/11 (F) 11/5

$$= \lim_{x\to\infty} \frac{5/x + 7/x^2 - 8/x^3 + 9/x^4}{11 + 7/x - 6/x^2 + 17/x^3 - 5/x^4}$$

(C)

6. [2 points] Find the limit $\lim\limits_{x\to 3} \dfrac{\sqrt{x^2+7}-4}{x-3}$. You must do this properly, showing all steps.

$$\lim_{x\to 3} \frac{\sqrt{x^2+7}-4}{x-3} = \lim_{x\to 3}\left(\frac{\sqrt{x^2+7}-4}{x-3}\right)\left(\frac{\sqrt{x^2+7}+4}{\sqrt{x^2+7}+4}\right)$$

$$= \lim_{x\to 3} \frac{(x^2+7)-16}{(x-3)(\sqrt{x^2+7}+4)}$$

$$= \lim_{x\to 3} \frac{x^2-9}{(x-3)(\sqrt{x^2+7}+4)}$$

$$= \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x^2+7}+4)} = \lim_{x\to 3} \frac{x+3}{\sqrt{x^2+7}+4} = \frac{6}{4+4} = \boxed{\frac{3}{4}}$$

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{5x^2}{x+2}$. Then verify your answer with the Quotient Rule.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{5(x+h)^2}{x+h+2} - \frac{5x^2}{x+2}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{5(x+2)(x^2+2xh+h^2) - 5x^2(x+h+2)}{(x+2)(x+h+2)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{5x^3} + \cancel{10x^2} + 10x^2h + 20xh + 5xh^2 + 10h^2 - \cancel{5x^3} - 5x^2h - \cancel{10x^2}}{(x+2)(x+h+2)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{5x^2h + 20xh + 5xh^2 + 10h^2}{(x+2)(x+h+2)}\right)$$

$$= \lim_{h\to 0} \frac{5x^2 + 20x + 5xh + 10h}{(x+2)(x+h+2)} = \boxed{\frac{5x^2+20x}{(x+2)^2}}$$

QR: $$f'(x) = \frac{10x(x+2) - 5x^2(1)}{(x+2)^2} = \boxed{\frac{5x^2+20x}{(x+2)^2}}$$

(c)

8. [5 points] Find the first derivatives of the following functions.

(a) $f(x) = e^{-5x}\sin(x^2)$

$\boxed{f'(x) = -5e^{-5x}\sin(x^2) + 2x e^{-5x}\cos(2x)}$

(b) $g(t) = \sqrt{7t^2 - 5t + 11}$ $= (7t^2 - 5t + 11)^{1/2}$

$\boxed{g'(t) = \frac{1}{2}(7t^2 - 5t + 11)^{-1/2}(14t - 5)} = \boxed{\frac{14t - 5}{2\sqrt{7t^2 - 5t + 11}}}$

(c) $\varphi(\theta) = \cot^2(2e^\theta)$

$\boxed{\varphi'(\theta) = 2\cot(2e^\theta)(-\csc(2e^\theta)\cot(2e^\theta))(2e^\theta)}$

$= \boxed{-4e^\theta \cot^2(2e^\theta)\csc(2e^\theta)}$

(d) $p(t) = 4^{\sqrt{t}}$

$\boxed{p'(t) = 4^{\sqrt{t}} \ln 4 \left(\frac{1}{2}t^{-1/2}\right)} = \boxed{\frac{1}{2\sqrt{t}} \ln 4 \; 4^{\sqrt{t}}}$

(e) $y = e^{x\sin(2x)}$

$\boxed{\frac{dy}{dx} = e^{x\sin(2x)}(\sin(2x) + 2x\cos(2x))}$