MAT 1322, Winter 2005

SAMPLE EXAMINATION

MAX = 46 points

This sample exam is last year's final exam slightly modified to fit this years's format. The correct answers of multiple choice problems are indicated by **bold-type**.

•Time: 3 hours
•Only TI 30-type calculators are permitted. Notes or books are not permitted.
•Problems 1-4 [4 points each] require detailed and clearly presented solutions. Show all work.
•Problems 5-19 [2 points each] are multiple choice. Circle the correct answer. Numerical answers are rounded to the given decimal.

•Work the problems in the space provided. Use the back-pages for rough work if necessary. A blank sheet for rough work is attached at the end of the questionnaire. Do not use any other paper. Work on multiple choice and answer-box problems will be examined only in case of suspected fraud.

•A blank sheet for rough work is attached at the end of the questionnaire.

1. [4 points] (a)[1 point] Sketch the region R of the plane enclosed by the curves $y = \sqrt{x}$ and $y = x$.
(b)[2 points]. Find the volume of the solid obtained by rotating the region R around the line $y = 2$. Sketch the solid showing a typical slice you use to set up the integral.

2. [4 points] A tank initially contains 800 litres of pure water and two pipes are connected to the tank. Through one pipe a solution enters the tank at the rate of 10 litres per minute which contains 14 grams of salt per litre and through the other pipe liquid leaves the tank at the rate of 10 litres per minute. Assume that the liquid in the tank is mixed instantaneously and is perfectly homogeneous.
(a)[2 points] Set up the differential equation for the quantity $Q(t)$ of salt (in grams) that is contained in the tank at time t and solve it to find $Q(t)$.
(b)[1 point] What is the amount of salt in the tank at time 40 minutes? What is the limiting concentration (in grams per litre) of salt in the tank?

(Provide a detailed solution and *draw a box around each answer.*)

3. [4 points] Let $f(x, y) = \cos(\pi\sqrt{x^2 + y^2})$.
(a)[1 point] Find the equation of the tangent plane to the graph of f at the point where $x = 0$ and $y = \frac{1}{4}$.
(b)[1 point] Sketch the contour map of f for $0 \leq \sqrt{x^2 + y^2} \leq \frac{1}{2}$ showing the level curves $f(x, y) = c$ for $c = 0, \frac{1}{2}, 1$. Label each curve by the value of c and label its intercept on the x-axis by the value of the x-coordinate.
(c)[1 point] Sketch the graph of f for $0 \leq \sqrt{x^2 + y^2} \leq \frac{1}{2}$. Start by sketching its trace in the plane $x = 0$.

4. [**4 points**] Let $f(x) = \frac{x^2}{1+x^6}$.
(a)Find the Taylor series of function $f(x)$ about $x = 0$ and its radius of convergence R.
(b)Use part (a) to find the Taylor series of $\arctan x^3$ about $x = 0$.

5. [**2 points**] Find the area of the region enclosed by the curves $x = 4 - y^2$ and $x = y^2 - 4$.

A. 21 B. 39/2 C. 37/3 D. 42/3 **E. 64/3** F. 83/4

6. [**2 points**] Find the arclength of the curve $x = t^2$, $y = 2t^3$ between the points $(0, 0)$ and $(1, 2)$.

A. 2.268 B. 2.295 C. 2.312 D. 2.391 E. 2.427 F. 2.443

7. [**2 points**] Use Euler's method with step size 0.1 to estimate $y(0.2)$, where y is the solution of the initial value problem $y' = 4x + 5y$, $y(0) = 1$

A. 2.29 B. 9.4 C. 1.96 D. 2.26 E. 1.52 F. 2.19

8. [2 points] Find $y(\pi/2)$ if y is the solution of the initial value problem

$$\frac{dy}{dt} = y\cos t + 2\cos t,\ y(0) = 1.$$

A. 6.155 B. π C. 2.718 D. 8.155 E. 10.155 F. 0.178

9. [2 points] A cup of milk is taken out of the fridge at 8am. The room temperature is 20 $^\circ$C and the temperature in the fridge is 5 $^\circ$C. At 8:30am the temperature of the milk is 10 $^\circ$C. What is the temperature (in $^\circ$C) of the milk at 9:30am? (The rate of change of the temperature of an object is proportional to the temperature difference between the object and its surroundings.)

A. 15.6 B. 17.1 C. 16.7 D. 15.0 E. 13.7 F. 18.2

10. [2 points] Which of the following series are convergent?

$$\text{(a)} \sum_{n=0}^{\infty} \frac{n^3}{1+3^n} \qquad \text{(b)} \sum_{n=0}^{\infty} \frac{3^n}{1+n^3} \qquad \text{(c)} \sum_{n=1}^{\infty} \frac{1+(-1)^n n^2}{n^3}$$

A. a,b **B. a,c** C. b,c D. a only E. b only F. c only
G. all H. none

11. [2 points] Find the McLaurin series of $f(x) = \ln\left(\frac{1+x}{1-x}\right)$.

A. $\boldsymbol{\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}}$ B. $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{n}$ C. $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$
D. $\sum_{n=1}^{\infty} \frac{x^n}{n}$ E. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$ F. $\sum_{n=1}^{\infty} \frac{2(-1)^n x^{2n}}{n}$

12. [2 points] Determine which of the following functions has the power series expansion

$$\sum_{n=0}^{\infty}(-1)^n \frac{4^n x^{4n}}{(2n)!}.$$

A. $\sin(2x^2)$ B. e^{-4x} C. $\cos(2x)$ D. e^{-2x^2} **E.** $\cos\left(\mathbf{2x^2}\right)$ F. $\sin(2x^2)$

13. [2 points] Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty}(-1)^{n+1} \frac{(2x-1)^n}{n5^n}.$$

A. $(-2, 3)$ **B.** $\mathbf{(-2, 3]}$ C. $[-2, 3]$ D. $(-4, 6)$ E. $(-4, 6]$ F. $[-4, 6]$

14. [2 points] Find the directional derivative of the function

$$f(x, y, z) = x^2 + 3xy + 2z$$

at the point $(2, 0, -1)$ in the direction of the vector v= $\langle 2, 1, -2 \rangle$.

A. 5.77 B. -4.24 C. 2 D. 6.24 **E. 3.33** F. 1.23

15. [2 points] Suppose $w = f(u, v)$, $u = g(s, t)$, and $v = h(s, t)$ satisfy
$u(1, 1) = 2$, $u_s(1, 1) = -1$, $u_t(1, 1) = 4$
$v(1, 1) = 2$, $v_s(1, 1) = 3$, $v_t(1, 1) = 5$
$f_u(2, 2) = -2$, $f_v(2, 2) = 10$
Find $\frac{\partial w}{\partial s}$ when $(s, t) = (1, 1)$

A. -30 **B. 32** C. 28 D. 42 E. 24 F. -16

16. [2 points] Given $xy^3 + yz^3 + zx^3 = 3$. Find $\frac{\partial z}{\partial y}$ when $(x, y, z) = (1, 1, 1)$ by implicit differentiation.

A. 4 **B. −1** C. 1 D. −1.33 E. −0.75 F. impossible

17. [2 points] Use the linear approximation of the function

$$f(x, y) = \sqrt{x^2 + y^3}$$

at the point $(1, 2)$ to approximate $f(1.4,\ 1.6)$.

A. 3.033 B. 2.974 C. 2.611 **D. 2.333** E. 2.221 F. 2.061

18. [2 points] Determine which of the following functions $f(x, y)$ satisfy

$$f_{xx} + f_{yy} = 0.$$

(a) $e^x \sin y\ + e^y \cos x$,
(b) $x \sin y + y \cos x$,
(c)$\ln(x^2 + y^2)$.

A. a,b   **B. a,c**   C. b,c   D. a only   E. b only   F. c only
G. all   H. none

19. [2 points] Determine which of the following integrals are convergent

$$\text{(a)} \int_1^\infty \frac{dx}{x + 2e^x}, \qquad \text{(b)} \int_1^\infty \frac{dx}{2e^x}, \qquad \text{(c)} \int_1^\infty \frac{dx}{x}.$$

A. a,b   B. a,c   C. b,c   D. a only   E. b only   F. c only
G. all   H. none

Extra workspace. [This sheet may be detached if used for rough work only. Do NOT hand in the detached sheet.]