Discrete Structures I
COMP1805

Logical Expression Translation

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Additional Operations in Propositional Logic

the "Exclusive Disjunction" of Two Propositions

is Denoted $A \oplus B$ and read as "A or B but not both"

$A \oplus B$ has a value of True when
Either A or B (but Not Both) have a value of True

Otherwise, $A \oplus B$ has a value of False

Additional Operations in Propositional Logic

the Exclusive Disjunction $A \oplus B$ is False when Both A and B have the Same Value Otherwise (i.e., $A \neq B$), $A \oplus B$ has a value of True

A	B	$A \oplus B$	
T	T	F	← False
T	F	T	← True
F	T	T	← True
F	F	F	← False

Exclusive (and Inclusive) Disjunction

in English, the Word "Or" is used for communicating

Both Disjunction and Exclusive Disjunction

e.g., *A flipped coin can land on heads or tails.*

Exclusive Disjunction

e.g., *You like computers or you need this credit.*

(Inclusive) Disjunction

Additional Operations in Propositional Logic

the "Implication" of Two Propositions

is Denoted $A \rightarrow B$ and read aloud as "if A then B"

$A \rightarrow B$ has a value of False only when
A is True and B is False

Otherwise, $A \rightarrow B$ has a value of True

Additional Operations in Propositional Logic

the Implication $A \rightarrow B$ is False only when
A is True and B is False
Otherwise $A \rightarrow B$ has a value of True

A	B	$A \rightarrow B$	
T	T	T	← False
T	F	F	← True
F	T	T	← True
F	F	T	← False

Implication

Implication is probably the most Difficult Operator to Reconcile with the English language

in an Implication statement $P \rightarrow Q$

P is the "Antecedent" and Q is the "Consequent"

and is Read Aloud as "If P Then Q" or "P Only If Q"

Implication

the statement "If A Then B" Asserts that it is Safe to Conclude B if you are Certain that A is True

it Asserts Absolutely Nothing if A is False
so it Also Must be Considered True when A is False

Implication Example

If you have a Canadian Passport,
then you are a Canadian Citizen.

A represents ***You have a Canadian Passport.***

B represents ***You are a Canadian Citizen.***

What are All Possible Pairs of Truth Values for these Propositions?

Implication Example

maybe you have Canadian Passport and you are a Canadian Citizen

(i.e., A = True, B = True)

If you have a Canadian Passport, then you are a Canadian Citizen.

is Obviously True for this case

Implication Example

maybe you do Not have a Canadian Passport
but you are a Canadian Citizen
(i.e., A = False, B = True)

maybe you do Not have a Canadian Passport
and you are Not a Canadian Citizen
(i.e., A = False, B = False)

If you have a Canadian Passport,
then you are a Canadian Citizen.

is Obviously Also True for these cases

Implication Example

it is Definitely Not the case that you have a Canadian Passport And you are Not a Canadian Citizen

Implication Example

Alternatively, under what Conditions would

If you have a Canadian Passport,
then you are a Canadian Citizen.

be a Lie?

if there was One or More people who
Have Passports but were Not Citizens
(i.e., A = False, B = True)

Implication Statements

it can also be Difficult to Contrast (in English)
"If", "Only If", and "If and Only If"

B If A	$A \rightarrow B$
B Only If A	$A \leftarrow B$
B If and Only If A	$A \leftrightarrow B$

Implication Statements

I will eat something if there is salt on it.
There is salt on it. → *I will eat it.*

(this means that Anything with Salt would be Eaten)

I will eat something only if there is salt on it.
I will eat it. → *There is salt on it.*

(this means that Everything Eaten had Salt on it)

Implication Statements

for the Implication Statement $P \rightarrow Q$ ...

the "Inverse" is	$\neg P \rightarrow \neg Q$
the "Converse" is	$Q \rightarrow P$
the "Contrapositive" is	$\neg Q \rightarrow \neg P$

the Contrapositive is Both Inverted and Converted
and it is the Only One of the three that is
Equivalent to the Implication Statement $P \rightarrow Q$

Implication and Biconditional

the "Biconditional" of Two Propositions

is Denoted $A \leftrightarrow B$ and read as "B if and only if B"

$A \leftrightarrow B$ has a value of False when
Either A or B (but Not Both) have a value of True

Otherwise, $A \leftrightarrow B$ has a value of True

alternatively, $A \leftrightarrow B$ is True if A and B are the Same

Implication and Biconditional

the Biconditional $A \leftrightarrow B$ is True when
Both A and B have the Same Value
Otherwise (i.e., $A \neq B$), $A \leftrightarrow B$ has a value of False

A	B	$A \leftrightarrow B$	
T	T	T	← True
T	F	F	← False
F	T	F	← False
F	F	T	← True

Implication and Biconditional

the English terms Necessity and Sufficiency are used to communicate Implication and Biconditional

if N then S
S implies N
N is true whenever S is

} **Equivalent**

N is a Necessary Condition for S
S is a Sufficient Condition for N

Translation Exercises

Translate this Assertion into a Logical Expression

If it is raining in the morning then I will wear my hat and my coat when I leave for work.

R	***It is raining.***
H	***I will wear my hat.***
C	***I will wear my coat.***

Translation Exercises

If it is raining in the morning then I will wear my hat and my coat when I leave for work. $R \rightarrow (H \wedge C)$

Translation Exercises

If it is raining in the morning then I will wear my hat and my coat when I leave for work. $R \rightarrow (H \wedge C)$

R	H	C	$H \wedge C$	$R \rightarrow (H \wedge C)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

If it Rains and I do Not wear Coat And Hat, I was lying...

...but if it does Not Rain, then I can wear Coat And Hat Or Not (without lying)

Translation Exercises

Translate this Assertion into a Logical Expression

Regular attendance of the lectures is necessary for passing this course.

A	*You attend lectures.*
P	*You pass the course.*

Translation Exercises

Regular attendance of the lectures is necessary for passing this course. $P \rightarrow A$

Translation Exercises

Regular attendance of the lectures is necessary for passing this course. $P \rightarrow A$

A	P	$P \rightarrow A$
T	T	T
T	F	T
F	T	F
F	F	T

if you Pass the Course you Must have Attended, but if you do Not Pass you May Or May Not have Attended