Concordia University

Applied Ordinary Differential Equations, ENGR 213

Final Exam

20 April 2009

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Evaluation out of 100. Only admissible calculators are allowed.
Time allotted: Three hours.

1. (15) (a) Find the general solution of the differential equation

$$\frac{dy}{dx} = \left(\frac{2y+3}{4x+5}\right)^2 .$$

You may leave the solution in implicit form.

(b) Solve the initial value problem

$$x\frac{dy}{dx} + y = e^x , \qquad y(1) = 2$$

2. (10) Find the general solution (explicit or implicit) of the equation

$$(y^2 \cos x - 3x^2 y - 2x)\, dx + (2y \sin x - x^3 + \ln y)\, dy = 0 .$$

3. (10) Find the general solution of the equation using an appropriate substitution:

$$\frac{dy}{dx} = \tan^2(x+y) .$$

You may leave the solution in implicit form.

4. (10) The Space Shuttle lands in Kennedy Space Center. The spacecraft touches down at $t = 0$ with a velocity of 100 m/sec. The spacecraft chute is deployed at $t = 4$ sec. Between touch down and deployment of chute ($0 \le t \le 4$), the velocity of the spacecraft $V(t)$ (in m/sec) is governed by:

$$\frac{dV}{dt} = 0$$

and after the deployment of chute by:

$$\frac{dV}{dt} = -0.002V^2$$

Determine when the spacecraft velocity reaches 20 m/sec.

5. (10) Find the general solution of the following differential equations using the method of *undetermined coefficients*.

(a) $y'' + 6y' + 8y = \sin 3x$

(b) $y'' + 10y' + 25y = e^x$

6. (10) Find the general solution of the differential equation

$$2x^2y'' + 5xy' + y = x^2 - x$$

by *variation of parameters.*

7. (12) Solve the following system of differential equations by any method you wish (systematic elimination, undetermined coefficients, variation of parameters, or diagonalization).

Matrix.

$$\frac{dx}{dt} = 2x + 3y - e^{2t}$$
$$\frac{dy}{dt} = -x - 2y + e^{2t}.$$

8. (11) Find the power series solution about the ordinary point $x = 0$ for the initial value problem

$$y'' - 3xy' - y = 0, \quad y(0) = 1, \ y'(0) = 0.$$

$y = \sum_{n=0}^{\infty} c_n x^n$.

It suffices to give only the constant term and those of x, x^2 and x^3.

9. (12) Given the LRC-circuit with $L = \frac{5}{3}$ henries, $R = 10$ ohms, $C = \frac{1}{30}$ farads, and $E(t) = 50\cos t$ volts, the charge $q(t)$ satisfies the linear second order ordinary differential equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t).$$

(a) Find the charge $q(t)$ if $q(0) = 100$ coulombs and $q'(0) = 0$ amperes.

(b) Identify in $q(t)$ the transient terms and, respectively, the steady state terms. Is the circuit overdamped, underdamped, or critically damped?

distinct : overdamped
complex : underdamped
repeated :

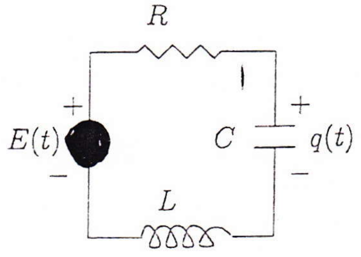

Figure 1: Problem 9.

Midterm Exam II ENGR 213
Applied Ordinary Differential Equations

Fall 2010 **November 22, 2010** **Time allowed: 1h 30min**

Name **ID**

Note: This is a closed-book examination. Only calculators approved by department are allowed. Please fill your name and student ID in the space provided above, and return the question paper with your examination booklet(s).

[10 points] **Problem 1.**
Given equation

$$y^{(4)} + y'' = 0,$$

verify that the functions

$$y_1(x) = 1, \quad y_2(x) = x, \quad y_3(x) = \cos x, \quad y_4(x) = \sin x$$

form a fundamental set of the solutions on interval $(-\infty, \infty)$. If so, write the general solution of the given equation.

[10 points] **Problem 2.**
Solve the following initial-value problem

$$y'' + 4y = 3\sin(2x), \quad y(0) = 1, \quad y'(0) = 0.$$

[10 points] **Problem 3.**
Solve the differential equation

$$y'' - 2y' + y = \frac{e^x}{x}.$$

[10 points] **Problem 4.**
Solve the following boundary-value problem

$$y''(e^x + 1) + y' = 0, \quad y(0) = 0, \quad y'(2) = 1.$$

See reverse side $\longrightarrow$

[10 points] **Problem 5.**
Given the equation of free undamped motion

$$\frac{d^2x}{dt^2} + 36x = 0, \quad x(0) = 4, \quad x'(0) = -18,$$

find the amplitude and the phase angle of free vibrations.

[5 points] **Bonus question.**
Find the general solution of the following nonlinear differential equation

$$y^3y'' = 1.$$

Midterm Exam I ENGR 213

Applied Ordinary Differential Equations

Fall 2010 October 25, 2010 Time allowed: 1h 15min

[10 points] **Problem 1.**

For each of the following first-order differential equations, state whether it is separable, linear, homogeneous, or none of the above. You do not need to explain your answer.

Each equation may correspond to more than one listed type (in which case you must list all types) or none of them

(a) $\sin x \cdot y' + \tan x \cdot y - e^x = 0$;

(b) $\sqrt{y^2+1}dx = xydy$;

(c) $y' = y^2 - \frac{2}{x^2}$;

(d) $(y^2 - 2xy)dx + x^2 dy = 0$;

(e) $\frac{dy}{dx} = \frac{xy}{y^2+4}$.

[10 points] **Problem 2.**

Find the general solution of the first-order linear differential equation

$$xy' - 2y = 2x^4.$$

Problem 3.

Given the differential equation

$$(1 + y^2 \sin(2x))dx + 2y \sin^2(x)dy = 0.$$

(a) [5 points] Verify that this is an exact differential equation.

(b) [5 points] Solve the equation leaving the general solution in implicit form.

See reverse side →

Problem 4.

The half-life of cesium-137 is 30 years. A sample originally has a mass of 100mg.

(a) [5 points] Find the mass of cesium-137 that remains after t years, if the rate of disintegration is proportional to the amount remaining.

(b) [5 points] After how long will only 1mg remain?

[5 points] **Bonus question.**

Given complex number $z = 8 + 8\sqrt{3}i$. Find $\sqrt[4]{z}$.

ENGR 213

APPLIED ORDINARY DIFFERENTIAL EQUATIONS

Midterm Examination II (FA) Monday March 26, 2012

Please attempt all problems (**1-3**). They have equal value.
Materials allowed: non-programmable calculators.

Problem 1

(1) Determine a second solution y_2 to the differential equation:

$$4t^2\frac{d^2y}{dt^2} + 8t\frac{dy}{dt} + y = 0, \quad t > 0$$

knowing that $y_1 = t^{-\frac{1}{2}}$ is a solution.
(2) Show that the two solutions y_1 and y_2 are linearly independent.

Problem 2

Solve the following differential equation:

$$y'' + 5y' + 6y = 4xe^{-2x} + e^{-2x}$$

Problem 3

Solve the following differential equation:

$$x^2y'' - 3xy' + 13y = 4 + 3x, \quad x > 0$$

Midterm Exam I ENGR 213
Applied Ordinary Differential Equations

Fall 2010 October 25, 2010 Time allowed: 1h 15min

[10 points] **Problem 1.**

For each of the following first-order differential equations, state whether it is separable, linear, homogeneous, or none of the above. You do not need to explain your answer.

Each equation may correspond to more than one listed type (in which case you must list all types) or none of them

(a) $\frac{dy}{dx} = (x-1)(x+1)$;

(b) $y' = y\sqrt{1-\frac{y}{x}}$;

(c) $\frac{dy}{dx} = y\sqrt{1-x^2}$;

(d) $x + 3y - xy' = 0$;

(e) $x\frac{dy}{dx} = \sqrt{x^2-y^2}$.

[10 points] **Problem 2.**

Find the general solution of the first-order linear differential equation

$$y' + 2y = e^{2x}.$$

Problem 3.

Given the differential equation

$$(\sqrt{x} + \frac{y}{x})dx + (y^2 + \ln(2x))dy = 0, \quad x > 0.$$

(a) [5 points] Verify that this is an exact differential equation.

(b) [5 points] Solve the equation leaving the general solution in implicit form.

See reverse side →

Problem 4.

Suppose that $A(t)$ (with t in months), the fish population in a lake contaminated by chemicals, satisfies the differential equation

$$\frac{da}{dt} = -kA(t), \quad k > 0.$$

(a) [5 points] Find the general solution of this differential equation.

(b) [5 points] Suppose today there are 500 fish in the lake, and we know that in month there will be 400 fish in the lake. How many fish will be there 1 year?

[10 points] **Problem 5.**

Given complex number $z = 8 + 8\sqrt{3}i$. Find $\sqrt[4]{z}$.

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APPLIED ORDINARY DIFFERENTIAL EQUATIONS

Midterm Test I February 13, 2012 Prof. Mariana Frank

SHARP EL-531 or CASIO FX-300MS calculators allowed, but no other materials.

Duration of the test: 1 hour, 15 minutes.

Problem 1.

Solve the following differential equations:

$$\textbf{(a)}\quad xy' + y = y^2, \quad y(1) = 2$$

$$\textbf{(b)}\quad \cos(x+y)dx + \left(3y^2 + 2y + \cos(x+y)\right) dy = 0$$

$$\textbf{(c)}\quad (x + y + 1)\, dx = (x + y - 1)\, dy$$

Problem 2.

A beaker of water initially at 50^0 C is placed to cool in a large bath of water. If in 10 minutes, the beaker cools to 30^0 C and in another 10 minutes to to 20^0 C, find the temperature of the bath of water. Assuming the bath is large and its the temperature stays approximately constant during this time.

QUICK REFERENCE CARD

to accompany Stewart's *Calculus*

Integration Formulas

Basic Formulas

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

2. $\int \frac{1}{x}\,dx = \ln|x| + C$

3. $\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

4. $\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

Trigonometric Functions

5. $\int \sin x\,dx = -\cos x + C$

6. $\int \cos x\,dx = \sin x + C$

7. $\int \sec^2 x\,dx = \tan x + C$

8. $\int \csc^2 x\,dx = -\cot x + C$

9. $\int \sec x \tan x\,dx = \sec x + C$

10. $\int \csc x \cot x\,dx = -\csc x + C$

11. $\int \tan x\,dx = \ln|\sec x| + C$

12. $\int \cot x\,dx = \ln|\sin x| + C$

13. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

14. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$

15. $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

16. $\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

17. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

18. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

Exponential and Logarithmic Functions

19. $\int e^x\,dx = e^x + C$

20. $\int a^x\,dx = \frac{a^x}{\ln a} + C$

21. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2 + b^2} + C$

22. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2} + C$

23. $\int xe^{ax}\,dx = \frac{1}{a^2}(ax - 1)e^{ax} + C$

24. $\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$

25. $\int \ln x\,dx = x\ln x - x + C$

26. $\int x^n \ln x\,dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + C$

Radicals

27. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$

28. $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{2} + C$

29. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln(x + \sqrt{a^2 + x^2}) + C$

30. $\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ln(x + \sqrt{a^2 + x^2}) + C$

31. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$

32. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln|x + \sqrt{x^2 - a^2}| + C$

33. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + C$

34. $\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

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