

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Duration
Midterm	8 March, 2015	1 h 30 min
Special Instructions:	Only approved calculators are allowed	
	Show all your work for full marks	

1. (10 marks): (a) Graph the function $f(x) = \begin{cases} -x - 1 & \text{if } x \leq 0 \\ -\sqrt{1 - x^2} & \text{if } x > 0 \end{cases}$, and use it

to calculate $\int_{-3}^1 f(x) dx$ in terms of area.

(b) Let for continuous functions f and g we know $\int_{-3}^0 f(x) dx = 2$, $\int_{-3}^5 f(x) dx = 6$,
 $\int_0^5 g(x) dx = 4$. Calculate $\int_0^5 [3 - 2f(x) + 5g(x)] dx$, or explain why it is impossible.

2. (6 marks): Use the Fundamental Theorem of Calculus to find the derivative of the function

$$F(x) = \int_x^{x^2} e^{\sin t} dt$$

3. (10 marks): Calculate the following indefinite integrals

$$(a) \int \frac{x}{x^2 + 3x + 2} dx \quad (b) \int \cos x \cos^3(\sin x) dx$$

4. (6 marks): Let $f(x) = \sqrt{x}$. Find a value c on the interval $[1,4]$ such that $f(c)$ is equal to the average value of $f(x)$ on that interval $[1,4]$.

5. (12 marks): Evaluate the following definite integrals (*do not approximate*):

$$(a) \int_0^2 \frac{x^5}{\sqrt{1 + 2x^2}} dx \quad (b) \int_0^\pi x^2 \sin(2x) dx$$

6. (6 marks): Sketch the graphs of functions $y = |x| - 2$ and $y = 4 - x^2$, and find the area of the region bounded by these graphs.

Bonus. (3 marks): Prove that for any function $f(x)$ integrable on the interval $[a, b]$ the right and left Riemann sums R_n and L_n are connected by the relation

$$R_n - L_n = \frac{f(b) - f(a)}{n}(b - a)$$