PART A (18 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE CODED ON THE SCANTRON SHEET. ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1 mark

1. If $n(A) = 25$, $n(B) = 47$ and $n(A \cap B^c) = 14$, find $n(A \cup B)$.

A: 58	B: 72	C: 61	D: 86	E: Cannot be determined.

1 mark

2. There are 2 left-handed girls and 21 right-handed boys in a classroom. If there are 19 girls and 5 left-handed students in the classroom, how many students are there in total in the classroom?

A: 43	B: 47	C: 42	D: 45	E: 40

1 mark

3. In how many different ways can \$8 in change be given to a customer, if only \$5 bills, \$2 coins and \$1 coins will be used? (*Note:* The change will be handed to the customer all at once.)

A: 8	B: 7	C: 3	D: 6	E: 10

1 mark

4. In a semi-annual report for the 25 students in a high school class, the progress of each student will be evaluated as Excellent, Very Good, Satisfactory or Poor. How many different reports are possible?

A: $\dfrac{25!}{21!}$	B: 100	C: $\binom{25}{4}$	D: 25^4	E: 4^{25}

1 mark

5. How many subsets of $\{a, b, c, d, e, f, g\}$ are there which contain at least one of the vowels, a and e?

A: 2^5	B: $2^6 + 2^6$	C: 2^2	D: $2^2 + 2^5$	E: $2^7 - 2^5$

1 mark

6. In how many ways can eight students take seats around a table so that Anastasia and Fiona (two of the students) are sitting opposite to one another?

A: 8!	B: 6!	C: 7!	D: $7! \times 2$	E: $6! \times 2$

1 mark 7. Diana has eight candles, each with a different scent. In how many ways can she put 3 of these candles in a row on her dining table?

A: 8^3	B: $8 \times 7 \times 6$	C: $\binom{8}{3}$	D: 24	E: 3!

1 mark 8. In how many different ways can all of 5 identical pencils be given to some of 9 children so that none of the children receives more than one pencil?

A: 9^5	B: 5^9	C: $\binom{9}{5}$	D: $\frac{9!}{4!}$	E: 45

1 mark 9. A test in a Math course has 18 multiple choice questions. Each question has 5 answer choices, of which only 1 is correct. In how many different ways can a student who answers all questions get exactly 15 of them correct?

A: 5^{18}	B: $\frac{18!}{3!15!}$	C: 4^3	D: $\frac{18!}{3!15!} \times 4^3$	E: 5^{15}

1 mark 10. In how many ways can 4 different History books and 7 different Philosophy books be arranged on a bookshelf if all books of the same subject must be together, side-by-side?

A: 1	B: $\frac{11!}{7!4!}$	C: 2!4!7!	D: 2	E: 4!7!

1 mark 11. In how many ways can 4 identical copies of the same History book and 7 identical copies of the same Philosophy book be arranged on a bookshelf if all copies of the same book must be together, side-by-side?

A: 1	B: $\frac{11!}{7!4!}$	C: 2!4!7!	D: 2	E: 4!7!

1 mark 12. In how many different ways can a poker player receive a hand of 5 cards which contains no Diamonds and exactly 2 Clubs?

A: $13 \times 12 \times \binom{26}{3}$	B: $\binom{13}{2} + \binom{26}{3}$	C: $\binom{13}{2} + \binom{39}{3}$
D: $\binom{13}{2} \times \binom{26}{3}$	E: $\binom{13}{2} \times \binom{39}{3}$	

1 mark 13. In how many different ways can a committee of 6 people be selected from 3 women and 8 men if there must be at least 2 women on the committee?

A: $\binom{3}{2}\binom{8}{4} + \binom{8}{3}$	B: $\binom{11}{5}$	C: $\binom{3}{2}\binom{8}{4}$
D: $\binom{11}{5} - \binom{3}{2}\binom{8}{4} - \binom{8}{3}$	E: $\binom{3}{2}\binom{9}{4}$	

1 mark 14. Which one of the following statements is **false**?

A: $\binom{12}{7} = \binom{12}{5}$	B: $\binom{15}{0} = 0$	C: $\binom{10}{10} = 1$	D: $\binom{18}{1} = 18$	E: $\binom{16}{15} = 16$

1 mark 15. A college has hired 11 new employees. In how many different ways can 4 be assigned to be library assistants, 4 be assigned to be secretaries and 3 be assigned to be lab supervisors, if each of the new employees has the skills to perform any of these 3 tasks?

A: $\dfrac{11!}{4!4!3!}$	B: $\dfrac{11!}{4!4!3!} \div 2!$	C: $\dfrac{11!}{4!4!3!} \div 3!$	D: $11!$	E: 1

1 mark 16. In how many ways can 9 different gifts be distributed among 3 identical bags so that each bag contains 3 gifts?

A: $\binom{9}{3\ 3\ 3}$	B: $(3!)^3$	C: $\binom{9}{3\ 3\ 3} \times 3!$	D: $(3!)^4$	E: $\binom{9}{3\ 3\ 3} \div 3!$

1 mark 17. Joseph keeps some bananas in a basket on his desk. Some days he eats one or more bananas at his morning break, but other days he decides to get a muffin and eat that instead of having any bananas. At the beginning of last week, he had 3 (identical) bananas in the basket, and by the end of the week, 5 days later, he had eaten all of them. He doesn't remember how many bananas he ate on which day, or how many days he got a muffin instead. In how many different ways might Joseph have eaten the 3 bananas? (That is, in how many different ways might the 3 bananas have been distributed among some of the 5 days, for eating?)

A: $\binom{5}{3}$	B: $\binom{7}{5}$	C: 3^5	D: $\binom{7}{3}$	E: 5^3

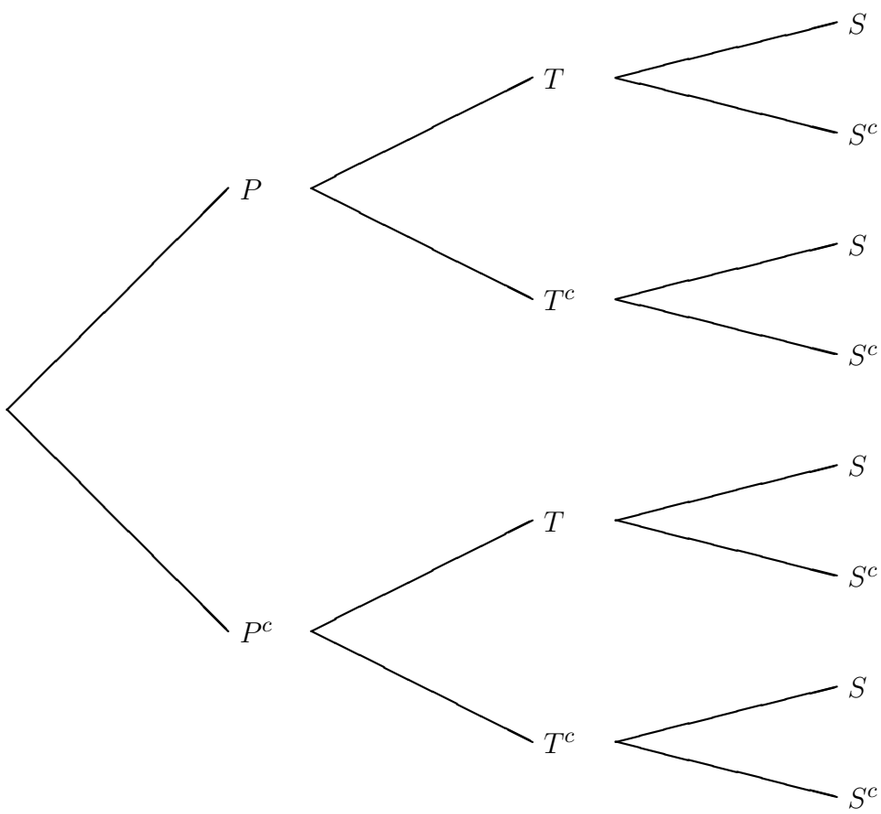
1 mark 18. In how many different ways can Dharla distribute 6 identical candies between her 4 children so that each child gets at least one candy?

A: $\binom{5}{2}$	B: $\binom{9}{6}$	C: $\dfrac{6!}{4!}$	D: $\binom{6}{4}$	E: 6^4

PART B (7 marks)

3 marks 19. 160 seniors participated in a survey on usage of technology. It was found that 130 of the participants have a personal computer (P), 60 participants have a tablet (T), and 40 have both. Also, 85 of the participants have a smart phone (S), of whom 5 also have both a personal computer and a tablet. Only 3 of the participants said they have no interest in technology and don't own any of the 3 kinds of electronic devices. And exactly 12 of the participants have only a tablet, but not either of the other 2 kinds of devices.

(a) Place the appropriate numbers on **all** the branches of the counting tree below.



(b) How many of the participants in the survey have exactly one of the three kinds of electronic devices?

2 marks 20. Ten friends have decided to spend Saturday evening playing board games. There are two board games available to them: *7-Wonders* and *Mysterium*. Since they cannot all play the same game, they decide that seven of them will play *7-Wonders* while the other three play *Mysterium*, but have not yet determined who will play which game. They are going to sit at two round tables of different sizes. In how many different ways can they **arrange** themselves with 7 at the larger table to play *7-Wonders* and 3 at the smaller table to play *Mysterium*?

Do not simplify your answer.

2 marks 21. How many different 5-letter sequences can be formed using the letters in the word *PAPAYA* if both the *P*s must be used, and the other letters can each be used at most the number of times it appears in the word *PAPAYA*?

Do not simplify your answer.