

School of Mathematics and Statistics
Carleton University
Math. 1004C, Fall 2016
TEST 5

Non-programmable calculators permitted as well as a few blank sheets but these should NOT be submitted.
Print Name :

Student Number:

Tutorial Section (C1, C2, C3, C4, or C5):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. [2 marks] Evaluate $\int_0^1 x^2 e^x dx$.

Solution. $\int_0^1 x^2 e^x dx = \int_0^1 x^2 de^x = e^x x^2 \Big|_0^1 - 2 \int_0^1 x e^x dx = e - 2 \int_0^1 x de^x = e - 2x e^x \Big|_0^1 + 2 \int_0^1 e^x dx = e - 2e + 2e^x \Big|_0^1 = -e + 2e - 2 = e - 2.$

(a) $2e - 2$, (b) $e^2 - 2$, (c) $2e - 1$, (d) $e - 2$, (e) none of these

2. [2 marks] Evaluate $\int_0^1 \ln(x+1) dx$.

Solution. $\int_0^1 \ln(x+1) dx = x \ln(x+1) \Big|_0^1 - \int_0^1 \frac{x}{x+1} dx = \ln 2 - \int_0^1 dx + \int_0^1 \frac{dx}{x+1} = \ln 2 - 1 + \ln(x+1) \Big|_0^1 = 2 \ln 2 - 1.$

(a) $e \ln 2$, (b) $2 \ln 2 - 1$, (c) $\ln 2 - 1$, (d) $\ln 2 - e$, (e) none of these

3. [2 marks] Find an antiderivative of $\frac{1}{x^2 + 4x + 5}$.

Solution. Completing the square $x^2 + 4x + 5 = (x+2)^2 + 1$, then $\int \frac{d(x+2)}{(x+2)^2 + 1} = \text{Arctan}(x+2) + C.$

(a) $\ln(x+2)$, (b) $\text{Arctan}(x+2)$, (c) $\ln(x+4)$, (d) $\text{Arccos}(x+4)$, (e) none of these

4. [2 marks] Evaluate $\int_0^{\pi/4} \sec^2 x \tan x dx$

Solution. Notice that $\sec^2 x dx = d \tan x$ then $\int_0^{\pi/4} \tan x d \tan x = \frac{\tan^2 x}{2} \Big|_0^{\pi/4} = \frac{1}{2}.$

(a) π , (b) $\frac{1}{2}$, (c) $\frac{\pi}{4}$, (d) 1, (e) none of these.

5. [2 marks] $\int_0^{\pi/2} \sin 2x \cos x dx = \frac{4}{3}.$

Solution. $I = \int_0^{\pi/2} \sin 2x d \sin x = \sin 2x \sin x \Big|_0^{\pi/2} - 2 \int_0^{\pi/2} \cos 2x \sin x dx = 2 \int_0^{\pi/2} \cos 2x d \cos x = 2 \cos 2x \cos x \Big|_0^{\pi/2} + 4 \int_0^{\pi/2} \sin 2x \cos x dx = -2 + 4I$, so $I = \frac{2}{3}$

OR using (i) $\sin 2x \cos x = 2 \sin x \cos^2 x$, OR (ii) $\sin 2x \cos x = \frac{1}{2}(\sin(3x) + \sin x)$

(a) TRUE, (b) FALSE,

PART II: Show all work here and give details.
No additional pages will be accepted

6. [6+4 marks] a) Evaluate $\int \frac{x}{(x^2+1)(x-1)} dx$

Solution. $\frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$ (1 point)

Cover-up: $C = \frac{1}{1+1} = \frac{1}{2}$ (1 point)

Plug-in $x = 0$: $0 = B - C$, so $B = \frac{1}{2}$ (1 point)

Plug-in $x = -1$: $\frac{-1}{2(-2)} = \frac{-A + 1/2}{2} + \frac{1}{2(-2)}$, so $A = -\frac{1}{2}$ (1 point)

$\frac{1}{2} \int \frac{-x+1}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx = -\frac{1}{4} \ln(x^2+1) + \frac{1}{2} \text{Arctan}x + \frac{1}{2} \ln|x-1| + C$ (2 points).

b) Evaluate $\int \frac{1}{x^2-1} dx$.

Solution. $\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$ (1 point)

Cover-up: $A = \frac{1}{1+1} = \frac{1}{2}$ (1 point)

Cover-up: $B = \frac{1}{-1-1} = -\frac{1}{2}$ (1 point)

$\frac{1}{2} \int \frac{1}{x-1} - \frac{1}{2} \int \frac{1}{x+1} dx = \frac{1}{2} (\ln|x-1| - \ln|x+1|) = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + C$ (2 points)

7. [5+5 marks]

a) Evaluate $\int_0^\pi \cos^2 x \sin^3 x dx$.

Solution. $\int_0^\pi \cos^2 x \sin^3 x dx = - \int_0^\pi \cos^2 x (1 - \cos^2 x) d \cos x = [u = \cos x] =$ (2 points)

$= - \int_1^{-1} u^2(1-u^2) du = \int_{-1}^1 (-u^4 + u^2) du = -\frac{u^5}{5} \Big|_{-1}^1 + \frac{u^3}{3} \Big|_{-1}^1 = -\frac{2}{5} + \frac{2}{3} = \frac{4}{15}$ (3 points)

b) Evaluate $\int e^{2x} \cos x dx$.

Solution. $I = \int e^{2x} d \sin x = e^{2x} \sin x - \int \sin x \cdot 2e^{2x} dx = e^{2x} \sin x + \int 2e^{2x} d \cos x =$ (2 points)

$= e^{2x} \sin x + 2 \cos x e^{2x} - 4 \int e^{2x} \cos x dx = e^{2x} (\sin x + 2 \cos x) - 4I$ (2 points),

so $I = \frac{e^{2x}}{5} (\sin x + 2 \cos x) + C$. (1 point)