**School of Mathematics and Statistics**
**Carleton University**
**Math. 1004C, Fall 2016**

# TEST 4

**Non-programmable calculators permitted** as well as a few blank sheets but these should NOT be submitted.
**Print Name :**

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**Student Number:**

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**Tutorial Section (C1, C2, C3, C4, or C5):**

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**PART I: Multiple Choice Questions**
(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Which of the following is an antiderivative of the function $f(x) = \dfrac{1}{1+x^2}$?

   (a) $\tan x$, $\boxed{\text{(b) Arctan}x}$, (c) Arcsin$x$, (d) $-$Arcsin$x$, e) none of these.

2. **[2 marks]** Evaluate $\displaystyle\int_0^{\pi/4} \sin 3x \sin x dx$.

   **Solution:** $\sin 3x \sin x = \dfrac{1}{2}(\cos(3x-x) - \cos(3x+x)) = \dfrac{1}{2}(\cos 2x - \cos 4x)$

   $$\int_0^{\pi/4} \sin 3x \sin x dx = \frac{1}{2}\int_0^{\pi/4} \cos 2x dx - \frac{1}{2}\int_0^{\pi/4} \cos 4x dx = \frac{1}{4}\sin 2x\Big|_0^{\pi/4} - \frac{1}{8}\sin 4x\Big|_0^{\pi/4} = \frac{1}{4} - 0 - 0 + 0 = \frac{1}{4}.$$

   (a) $\pi/2$, $\boxed{\text{(b) } 1/4}$, (c) 1, (d) 1/2, e) none of these

3. **[2 marks]** Find an antiderivative of $f(x) = 3^{x+1}$.

   **Solution:** $3^{x+1} = e^{\ln 3(x+1)}$, so $\mathcal{F}(x) = \dfrac{1}{\ln 3}e^{\ln 3(x+1)} = \dfrac{3^{x+1}}{\ln 3}$

   (a) $e^{x+1}$, (b) $3^{x+1}/2$, $\boxed{\text{(c) } 3^{x+1}/\ln 3}$, (d) $e^{x+1}/\ln 3$, (e) none of these.

4. **[2 marks]** Find the area under the curve defined by $y = 3x\sqrt{x^2+1}$ above the $x-$axis and between the vertical lines $x = 0$ and $x = 1$.

   **Solution:** $\displaystyle\int_0^1 3x\sqrt{x^2+1}dx = (x^2+1)^{3/2}\Big|_0^1 = 2^{3/2} - 1 = 2\sqrt{2} - 1.$

   (a) $\boxed{2\sqrt{2}-1}$, (b) $2\sqrt{2}-2$, (c) $\sqrt{2}-1$, (d) $2-\sqrt{2}$, (e) none of these.

5. **[2 marks]** $\displaystyle\int_0^{\pi} \sin^2 x = \frac{\pi}{4}$.

   **Solution:** $\sin^2 x = \dfrac{1-\cos 2x}{2}$, so $\displaystyle\int_0^{\pi} \sin^2 x = \frac{1}{2}\int_0^{\pi} dx - \frac{1}{2}\int_0^{\pi} \cos 2x dx = \frac{\pi}{2} - \frac{1}{4}\sin 2x\Big|_0^{\pi} = \frac{\pi}{2}.$

   (a) TRUE, $\boxed{\text{(b) FALSE}}$.

**PART II: Show all work here and give details.**
**No additional pages will be accepted**

6. **[5+5 marks]** a) Find the most general antiderivative of the function $\dfrac{x}{x^2+1}$.

   **Solution:** $\displaystyle\int \frac{x}{x^2+1}dx = \int \frac{1}{2}(\ln(x^2+1))'dx = \frac{1}{2}\ln(x^2+1) + C$

   (3*points*) (1*point*) (1*point*)

   b) Evaluate the limit $\displaystyle\lim_{x\to 0+} \frac{d}{dx}\int_1^x \frac{\sin t}{t}dt$.

   **Solution:** $\displaystyle\lim_{x\to 0+} \frac{d}{dx}\int_1^x \frac{\sin t}{t}dt = \lim_{x\to 0+} \frac{\sin x}{x}(x)' = 1.$ (4+1*points*)

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7. **[6+4 marks]**

a) Find the solution of the differential equation subject to the given initial condition. Live the solution in implicit form.

$y' = \dfrac{\sec^2 x}{\sin y}$, $y(0) = \pi$

**Solution:** $\sin y dy = \sec^2 x dx$ (*1point*). After integration: $-\cos y = \tan x + C$.(*2point*)
Apply IC: $-(-1) = 0 + C$, so $C = 1$(*1point*) and $\cos y = -\tan x - 1$(*1point*).
$y = \text{Arccos}(-\tan x - 1) = \pi - \text{Arccos}(\tan x + 1)$, and $0 \geq x \geq -\dfrac{\pi}{4}$ (*bonus 1 point*)

b) Evaluate $\displaystyle\int \frac{\ln(x+1)}{x+1}\, dx$.

**Solution:** substitution $u = x+1$, then $du = dx$ and $\displaystyle\int \frac{\ln(x+1)}{x+1}\, dx = \int \frac{\ln u}{u}\, du$ (1point). One more substitution: $t = \ln u$, $dt = du/u$ (1point). Then $\displaystyle\int t dt = \frac{t^2}{2} + C = \frac{(\ln u)^2}{2} + C = \frac{(\ln(x+1))^2}{2} + C$ (3 points).
*Remark.* At once to set $t = \ln(x+1)$ is also acceptable.