School of Mathematics and Statistics
Carleton University
Math. 1004C, Fall 2016

TEST 3

Non-programmable calculators permitted as well as a few blank sheets but these should NOT be submitted.
Print Name :

Student Number:

Tutorial Section (C1, C2, C3, C4, or C5):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** If $\log_2 x^2 = y$, and $x > 0$, what is x ?
   Solution. By the power property: $2\log_2 x = y$, so $\log_2 x = y/2$. Then, by definition of log: $x = 2^{y/2}$.

   (a) $x = \left(\frac{y}{2}\right)^2$, (b) $x = 2^{2/y}$, (c) $x = e^{y/2}$, $\boxed{\text{(d) } x = 2^{y/2}}$, e) none of these

2. **[2 marks]** Let $f(x) = 5^{x^2}$. Find $f'(1)$, that is find the derivative of f at $x = 1$.
   Solution. $f'(x) = (e^{\ln 5\cdot x^2})' = 2x\ln 5 \cdot 5^{x^2}$. Now $f'(1) = 2\ln 5 \cdot 5 = 10\ln 5$.

   (a) $2\log_5 e$, $\boxed{\text{(b) } 10\ln 5}$, (c) $5\ln 2$, (d) $\log_2 5$, e) none of these

3. **[2 marks]** Let $f(x) = \log_3(x^2 + x + 1)$. Find $f'(0)$, that is find the derivative of f at $x = 0$.
   Solution. $f'(x) = \left(\frac{\ln(x^2+x+1)}{\ln 3}\right)' = \frac{(x^2+x+1)'}{\ln 3(x^2+x+1)} = \frac{2x+1}{\ln 3(x^2+x+1)}$. Then $f'(0) = \frac{1}{\ln 3}$

   (a) $-\ln 3$, $\boxed{\text{(b) } 1/\ln 3}$, (c) 0, (d) $\ln 3$, e) none of these

4. **[2 marks]** Let $f(x) = \ln x - x^2 + x$. Then on the interval $(0, \infty)$ the function $f(x)$ has a
   Solution. $f'(x) = \frac{1}{x} - 2x + 1 = -\frac{2x^2 - x - 1}{x} = -\frac{(2x+1)(x-1)}{x}$. Setting $f'(x) = 0$ gives two roots: $x_1 = -\frac{1}{2}$ (but it is less than 0) and $x_2 = 1$, which we should check. $f''(x) = -\frac{1}{x^2} - 2$, so $f''(1) = -3 < 0$, i.e. a local maximum.

   (a) local minimum, $\boxed{\text{(b) local maximum}}$, (c) inflection point, (d) none of these

5. **[2 marks]** Let $f(x) = \frac{x^3}{6} - x^2 + x + \frac{2}{3}$. Then $f(x)$ has an inflection point at $(2, 0)$.
   Solution. $f'(x) = \frac{x^2}{2} - 2x + 1$, then $f''(x) = x - 2$. So, at the point $x = 2$ the second derivative of f changes its sign and $f(2) = 4/3 - 4 + 2 + 2/3 = 0$

   $\boxed{\text{(a) TRUE}}$, (b) FALSE,

PART II: Show all work here and give details.
No additional pages will be accepted

6. **[5+5 marks]** a) Let $f(x) = (x + 1)^x$. Evaluate $f'(0)$, that is find the derivative of f at $x = 0$.
   Solution. $f'(x) = \underbrace{\left(e^{x\ln(x+1)}\right)'}_{(2\ point)} = \underbrace{e^{x\ln(x+1)}\left(\ln(x+1) + \frac{x}{x+1}\right)}_{(2\ point)} = (x+1)^x\left(\ln(x+1) + \frac{x}{x+1}\right)$.

   $f'(0) = 1^0\left(\ln 1 + \frac{0}{1}\right) = 0.$(*1 point*)

   b) Let f be defined by $f(x) = \log_x(x + 1)$. Evaluate the derivative $f'(x)$ for $x > 0$.

   Solution. $f'(x) = \underbrace{\left(\frac{\ln(x+1)}{\ln x}\right)'}_{(2\ point)} = \underbrace{\frac{\frac{1}{x+1}\ln x - \frac{1}{x}\ln(x+1)}{(\ln x)^2}}_{(3\ point)}$.

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7. **[4+4+2 marks]** a) Solve the polynomial inequality $p(x) = (x-1)^2(x^2-x-6) > 0$ for $-\infty < x < +\infty$.
Solution. Notice that $p(x) = (x-1)^2(x-3)(x+2)$.(*1 point*)
Construct the SDT: (*2 point*)

x in	$(x+2)$	$(x-1)^2$	$(x-3)$	Sign of $p(x)$
$(-\infty,-2)$	$-$	$+$	$-$	$+$
$(-2,1)$	$+$	$+$	$-$	$-$
$(1,3)$	$+$	$+$	$-$	$-$
$(3,\infty)$	$+$	$+$	$+$	$+$

From the last column $p(x) > 0$ if x is in $(-\infty,-2)$ or $(3,\infty)$.(*1 point*)

b) Solve the rational function inequality $r(x) = \dfrac{x-2}{x^2+x-2} < 0$ for $-\infty < x < +\infty$.

Solution. Notice that $r(x) = \dfrac{x-2}{(x+2)(x-1)}$.(*1 point*)

Construct the SDT(*2 point*):

x in	$(x+2)$	$(x-1)$	$(x-2)$	Sign of $r(x)$
$(-\infty,-2)$	$-$	$-$	$-$	$-$
$(-2,1)$	$+$	$-$	$-$	$+$
$(1,2)$	$+$	$+$	$-$	$-$
$(2,\infty)$	$+$	$+$	$+$	$+$

From the last column $r(x) < 0$ if x is in $(-\infty,-2)$ or $(1,2)$ (*1 point*).
c) Find asymptotes of $r(x)$, see b), if any.
Solution. Vertical asymptotes, see the denominator: $x = -2$ and $x = 2$.(*1 point*)

Horizontal asymptotes: $\lim\limits_{x\to\pm\infty} \dfrac{x-2}{x^2+x-2} = 0$, so $y = 0$.(*1 point*)

There are no slant asymptotes as $m = \lim\limits_{x\to\pm\infty} \dfrac{x-2}{x(x^2+x-2)} = 0$.