

School of Mathematics and Statistics  
Carleton University  
Math. 1004C, Fall 2016  
**TEST 1**

Non-programmable calculators permitted as well as a few blank sheets but these should NOT be submitted.

Print Name :

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Student Number:

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Tutorial Section (C1, C2, ...):

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**PART I: Multiple Choice Questions**

(Choose and CIRCLE only ONE answer - No part marks here.)

1. [2 marks] Evaluate the limit:  $\lim_{t \rightarrow 0} \frac{1 - \cos 2t}{t^2}$ .

**Solution** (just for your information, you only need to circle an answer in part I):

$$\lim_{t \rightarrow 0} \frac{1 - \cos 2t}{t^2} = \lim_{t \rightarrow 0} \frac{2 \sin^2 t}{t^2} = 2 \left( \lim_{t \rightarrow 0} \frac{\sin t}{t} \right)^2 = 2 \cdot (1)^2 = 2.$$

(a) 1, (b) 0, (c) -1, (d) 2.

2. [2 marks] Let  $f(x) = \frac{1}{1 + \sin x}$ . Evaluate  $f'(\pi)$  using any method:

**Solution:**

$$f'(x) = -\frac{\cos x}{(1 + \sin x)^2}, \text{ so } f'(\pi) = -\frac{\cos(\pi)}{(1 + \sin(\pi))^2} = -\frac{-1}{(1 + 0)^2} = 1.$$

(a) -1, (b) 1, (c) 0, (d) 1/2.

3. [2 marks] Let  $f$  be defined by

$$f(t) = \begin{cases} 1 - \frac{t^2}{2}, & \text{if } t \leq 0, \\ \frac{1 + \cos 2t}{2}, & \text{if } t > 0. \end{cases}$$

Is  $f$  continuous at  $t = 0$ ?

**Solution:**

$$L_{\text{left}} = \lim_{x \rightarrow 0^-} \left(1 - \frac{t^2}{2}\right) = 1 - 0 = 1,$$

$$L_{\text{right}} = \lim_{x \rightarrow 0^+} \frac{1 + \cos 2t}{2} = \frac{1 + 1}{2} = 1,$$

$$f(0) = \left(1 - \frac{t^2}{2}\right) \Big|_{t=0} = 1.$$

So,  $L = L_{\text{left}} = L_{\text{right}} = f(0)$ .

(a) YES, (b) NO,

4. [2 marks] Evaluate the following limit,  $\lim_{x \rightarrow \infty} \frac{\cos x - \sin x}{x}$ .

**Solution:**

$$\cos x - \sin x = \sqrt{2} \cos(x + \pi/4), \text{ then by the Sandwich theorem } \lim_{x \rightarrow \infty} \frac{\cos(x + \pi/4)}{x} = 0$$

(a) -1, (b) 1, (c) 0, (d) The limit does not exist.

5. [2 marks] Let an implicit function is given by the equation  $x^3 + 2xy + y^2 = 1$ . Evaluate  $y'$  at the point  $(1, 0)$ .

**Solution:**

$$\text{By implicit differentiation: } 3x^2 + 2y + 2xy' + 2yy' = 0, \text{ so } y' = -\frac{3x^2 + 2y}{2(x + y)}, \text{ and } y'(1, 0) = -\frac{3}{2}$$

(a) -3/2, (b) 1/2, (c) 0, (d) -1/2.

**PART II: Show all work here.**  
**No additional pages will be accepted**

6. [5+5 marks] :

a)  $f(x) = \tan(\sin(\cos x))$ . Find the derivative,  $f'(x)$ .

b) Evaluate the following limit:  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$ .

**Solution:** a)

$$\begin{aligned} \frac{d}{dx}(\tan(\sin(\cos x))) &= [\tan(\sin(\cos x))]}' = \frac{1}{\cos^2(\sin(\cos x))} \cdot \cos(\cos x) \cdot (-\sin x) \\ &\quad \text{2 points} \quad \text{2 points} \quad \text{1 point} \\ \text{OR} \quad &= \sec^2(\sin(\cos x)) \cdot \cos(\cos x) \cdot (-\sin x) \end{aligned}$$

b)

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{(\sqrt{x^2 + x} + x)} = \lim_{x \rightarrow \infty} \frac{(x^2 + x - x^2)}{(\sqrt{x^2 + x} + x)} = \lim_{x \rightarrow \infty} \frac{x}{x(\sqrt{1 + 1/x} + 1)} = \\ &\quad \text{2 points} \quad \text{2 points} \\ &\quad \lim_{x \rightarrow \infty} \frac{1}{(\sqrt{1 + 1/x} + 1)} = \frac{1}{2} \\ &\quad \text{1 point} \end{aligned}$$

7. [5+5 marks]

a) Evaluate the following limit:  $\lim_{x \rightarrow 0} \frac{\tan 6x}{\tan 9x}$ .

b) Find the derivative,  $f'(x)$  of the function  $f$  defined by  $f(x) = \frac{x^2 + \sqrt{x}}{x + 2}$ .

**Solution:** a)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan 6x}{\tan 9x} &= \lim_{x \rightarrow 0} \frac{\sin 6x}{\sin 9x} \cdot \lim_{x \rightarrow 0} \frac{\cos 9x}{\cos 6x} = \lim_{x \rightarrow 0} \frac{\sin 6x}{6x} \frac{6x}{9x} \frac{9x}{\sin 9x} = \frac{6}{9} \lim_{x \rightarrow 0} \frac{\sin 6x}{6x} \lim_{x \rightarrow 0} \frac{9x}{\sin 9x} = \frac{2}{3} \\ &\quad \text{1 point} \quad \text{2 points} \quad \text{2 points} \end{aligned}$$

b)

$$\left( \frac{x^2 + \sqrt{x}}{x + 2} \right)' = \frac{\left( 2x + \frac{1}{2\sqrt{x}} \right)(x + 2) - x^2 - \sqrt{x}}{(x + 2)^2} = \frac{x^2 + 4x - \frac{\sqrt{x}}{2} + \frac{1}{\sqrt{x}}}{(x + 2)^2}.$$

4 points for the correct numerator and 1 point for the correct denominator; if simplified +1 bonus point