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CARLETON UNIVERSITY

FINAL
EXAMINATION
APRIL 2000

DURATION: 3.0 HOURS

No. of Students: 174

Department Name & Course Number: Mechanical and Aerospace Engineering 86.211B

Course Instructor(s): F.F. Afagh

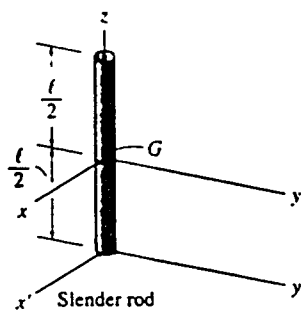
AUTHORIZED MEMORANDA

Calculators, rulers and protractors only.

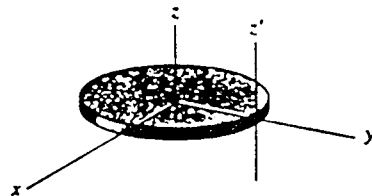
Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This question paper has **11** pages.

This examination question paper **MAY** be taken from the examination room.

- a) Answer not more than **FIVE** questions from parts A and B of this examination.
- b) The marks for each question are as indicated at the end of each question.
- c) On your answer book indicate **CLEARLY** the **FIVE** questions that you have answered and you wish to be marked. Otherwise the first five questions will be considered as answered and they will be marked.



$$I_{xx} = I_{yy} = \frac{1}{12} m \ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m \ell^2 \quad I_{zz} = 0$$



Thin circular disk

$$I_{xx} = I_{yy} = \frac{1}{4} m r^2 \quad I_{zz} = \frac{1}{2} m r^2 \quad I_{z'z'} = \frac{3}{2} m r^2$$

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PART A

- 1) A droptest machine is shown in Figure (1.a). The specimen is mounted on a rigid carriage, which upon release is dropped along frictionless guide rods onto a bumper spring and then onto a set of lead pads resting on a heavy rigid base. The bumper spring that has a $k = 5000 \text{ N/mm}$ helps to absorb a fraction of the energy of the carriage and specimen while it is deformed 10 mm. After that, the remaining energy of the carriage and specimen is assumed to be absorbed only by the 4 lead pads that are placed directly on the base. The energy E absorbed by a single pad versus its deformation δ is given as a parabolic relation shown in Fig. (1.b). The carriage and specimen together weigh 100 N.

Determine:

- a) the velocity at which the carriage impacts the lead pads;
- b) the deformation of each pad.

(20 points)

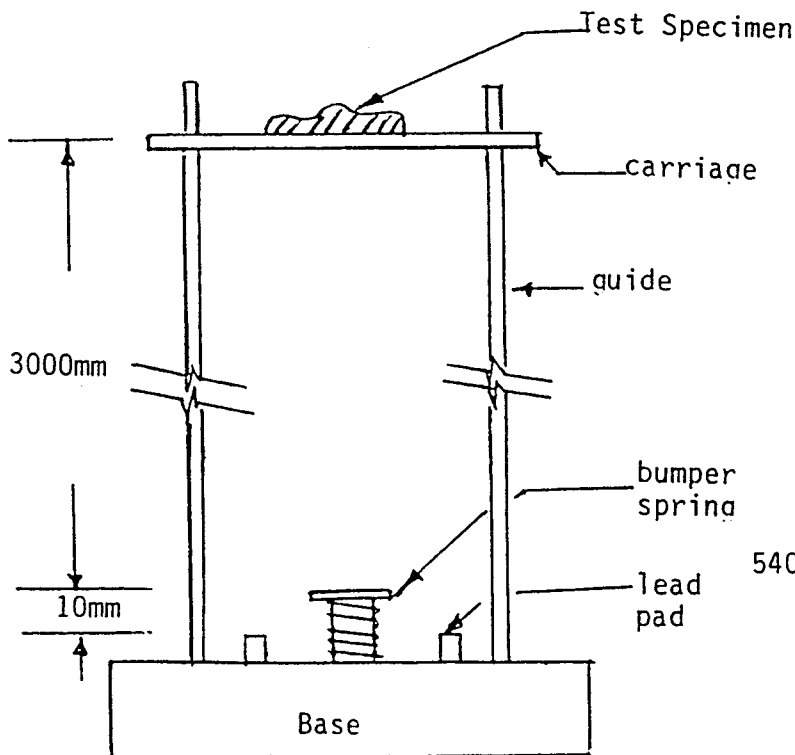


Figure (1-a)

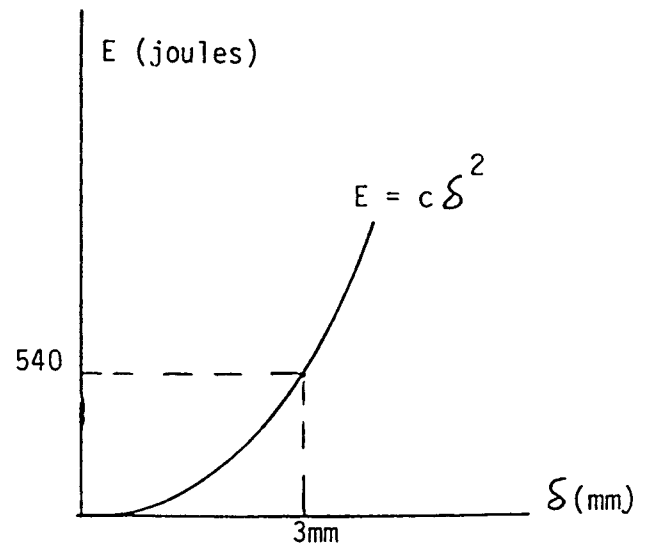


Figure (1-b)

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2) A mechanism with two sliders is shown. At the instant shown, slider A has a speed of 3 m/s to the right and is accelerating at the rate of 10 m/s^2 . The link AB is 2.5 m long.

Determine:

- a) the angular velocity of the link AB,
- b) the angular velocity of the slider B about O,
- c) the angular acceleration of the link AB.

Use Graphical method to solve this problem.

Suggestion: Velocity and acceleration diagrams may each be started in the middle of a page using the following scales:

velocity diagram: $10 \text{ mm} = 1 \text{ m/s}$

acceleration diagram: $10 \text{ mm} = 10 \text{ m/s}^2$

(20 points)

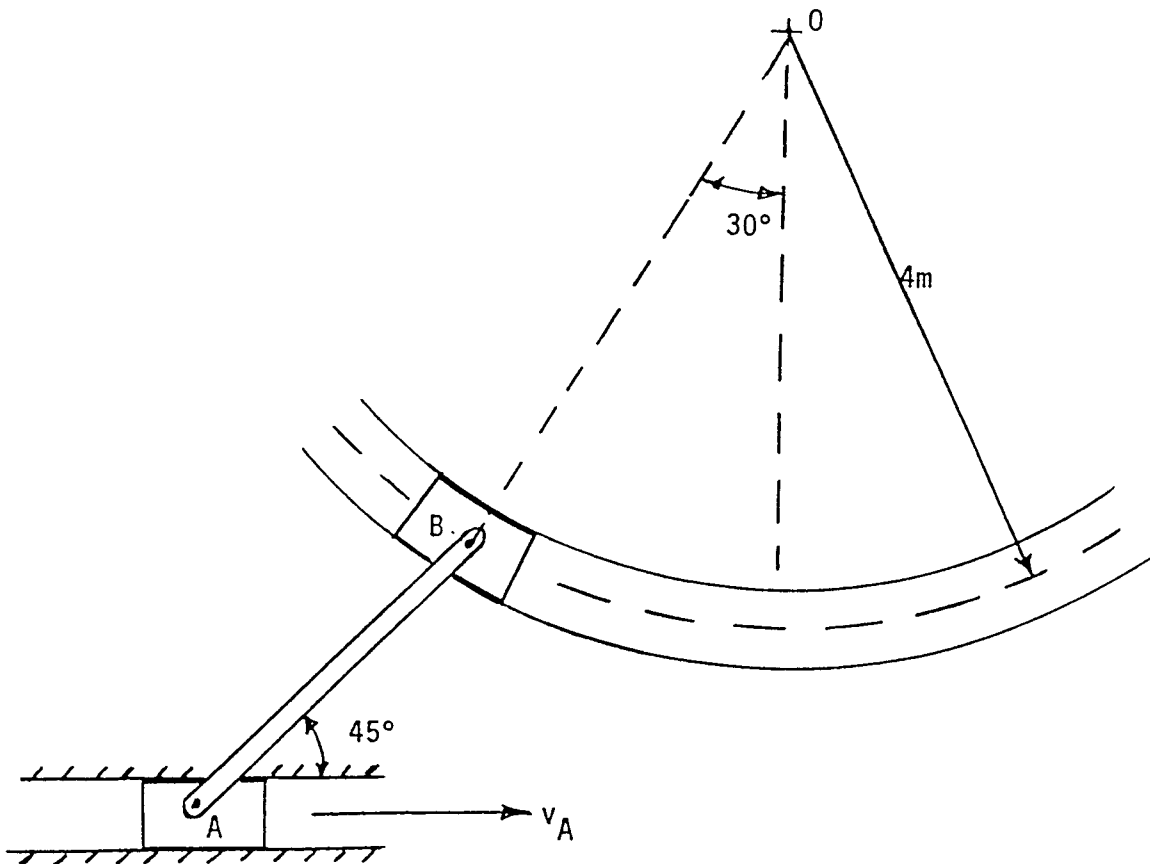


Figure 2

RZK
AJ

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- 3) Disk A has a weight of 20N and disk B has a weight of 40N. If no slipping occurs between them, determine the couple moment M which must be applied to disk A to give it an angular acceleration of 4 rad/s^2

(20 points)

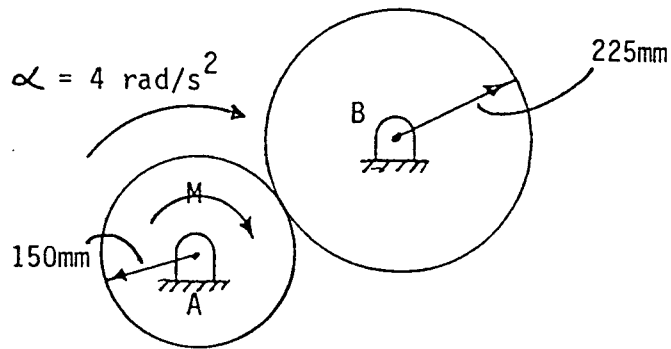


Figure 3

ELR

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- 4) The 20 N sphere is attached to a rod of negligible mass and rests in the horizontal position. Treating the sphere as a point mass, determine the natural frequency of vibration of the system.
(20 points)

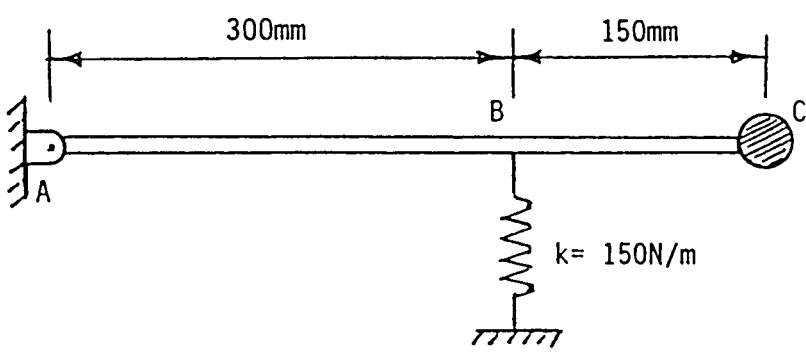


Figure 4

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PART B

- 5) The mass of the rocket sled is 1200 kg without fuel. The sled is initially at rest. Once the engine is ignited it produces a constant propulsive force of 10 kN for 12 s, during which time it burns up its 240 kg of fuel at a constant rate. All friction are negligible.

Determine:

- a) the speed of the sled when all the fuel has been used;
- b) how many seconds it will take for the sled to reach a velocity of 50 m/s;
- c) the acceleration of the sled when its velocity reaches 50 m/s

[Hint: in general, for a system with varying mass $F = \frac{d(mv)}{dt}$]

(30 points)

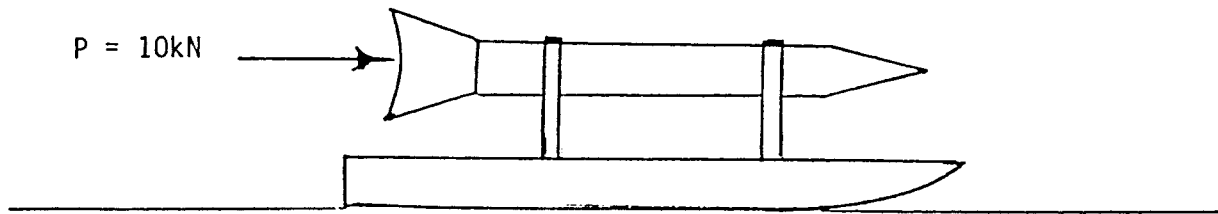


Figure 5

A/R
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- 6) The damper pivoted at C is operated by the hydraulic cylinder AB. At the instant shown, the piston rod is being extended with a velocity of 120 mm/s and an acceleration of 23.1 mm/s² relative to the cylinder.

For this instant, determine:

- a) the angular velocity of the damper ω_D , and
- b) the angular acceleration of the damper α_D

(30 points)

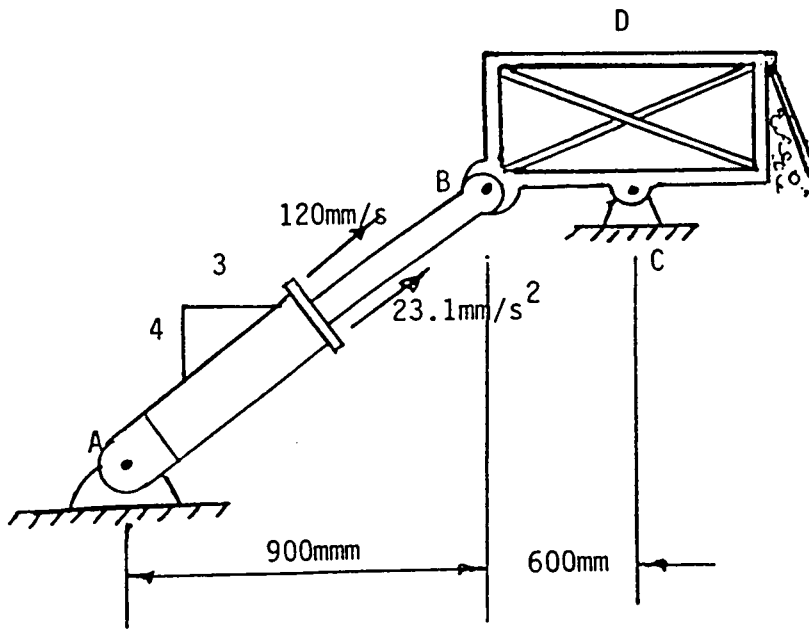


Figure 6

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- 7) The link AB and pivoted circular disk that has a constant clockwise angular velocity of 12 rad/s relative to the rod are released from rest in the position shown and swing in the vertical plane about the fixed bearing at A. The 4.5 kg link AB has a radius of gyration about A of 520 mm. The disk has a mass of 9 kg.

When the rod swings through the vertical position AB', determine:

- a) the angular velocity ω_{AB} of the rod, and
- b) the force F_A exerted at A on the link.

(25 points)

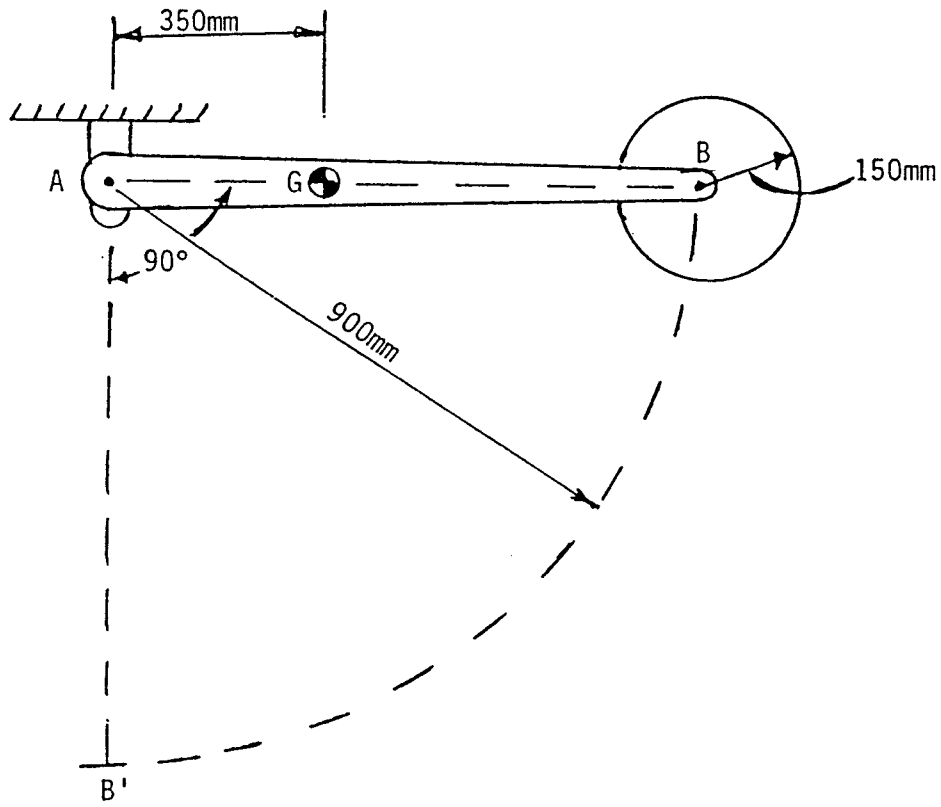


Figure 7

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- 8) The 1.5 kg sphere is moving at 2 m/s when it strikes the 3-kg stationary uniform slender rod B at point C. After the impact the velocity of the sphere is determined to be 0.8 m/s to the right.
- a) Determine the coefficient of restitution.
 - b) Is this mostly an elastic impact? Provide reason(s) for your answer.

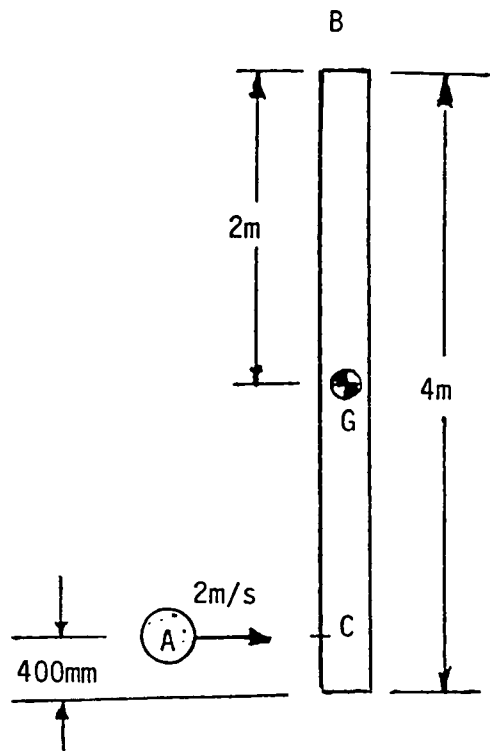
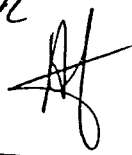


Figure 8

KAR


Fundamental Equations of Two Dimensional Dynamics

Particle Rectilinear Motion — Variable a

$$a = \frac{dv}{dt} \quad v = \frac{ds}{dt} \quad v dv = a ds$$

Particle Rectilinear Motion — Constant a

$$v = v_0 + at \quad s = s_0 + v_0 t + \frac{1}{2} at^2 \quad v^2 = v_0^2 + 2a(s - s_0)$$

Particle Curvilinear Motion — x, y Coordinates

$$v_x = \dot{x} \quad a_x = \ddot{x} \quad v_y = \dot{y} \quad a_y = \ddot{y}$$

Particle Curvilinear Motion — r, θ Coordinates

$$v_r = \dot{r} \quad a_r = \ddot{r} - r\dot{\theta}^2 \quad v_\theta = r\dot{\theta} \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Particle Curvilinear Motion — n, t Coordinates

$$v = \dot{s} \quad a_t = \dot{v} = \ddot{s} = v \frac{dv}{ds} \quad a_n = \frac{v^2}{\rho} \quad \rho = \left| \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} \right|$$

Relative Motion

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \quad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Rigid Body Rotation About a Fixed Axis — Variable α

$$\alpha = \frac{d\omega}{dt} \quad \omega = \frac{d\theta}{dt} \quad \omega d\omega = \alpha d\theta$$

Rigid Body Rotation About a Fixed Axis — Constant α

$$\omega = \omega_0 + \alpha t \quad \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Motion of a Point in Fixed Axis Rotation

$$s = r\theta \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$$

Motion Relative to Rotating Axes

$$\vec{v}_B = \vec{v}_A + \vec{\Omega} \times \vec{r}_{B/A} + \vec{v}_{B,rel}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A}) + 2\vec{\Omega} \times \vec{v}_{B,rel} + \vec{a}_{B,rel}$$

where $\vec{v}_{B,rel}, \vec{a}_{B,rel}$ — velocity/acceleration of point B relative to the rotating frame
 \vec{v}_A, \vec{a}_A — absolute velocity/acceleration of point A

Mass Moment of Inertia $I = \int r^2 dm$
 Parallel-Axis Theorem $I_o = I_G + md^2 = mk^2$
 Radius of Gyration $k^2 = I/m$

Uniform Bar $k^2 = L^2/12$ About G
 $k^2 = L^2/3$ About one end

Disc $k_D^2 = r^2/4$ Diametral Axis
 $k_P^2 = r^2/2$ Polar Axis

Sphere $k^2 = 2r^2/5$

Equations of Motion

Particle: $\sum \vec{F} = m\vec{a}$ Rigid Body: $\sum \vec{F} = m\vec{a}_C$
 $\sum \vec{M}_C = I_G \vec{\alpha}$

Other Moment Equations

$$\sum M_P = I_G \alpha + ma_C(d) \quad \text{or}$$

$$\sum \vec{M}_P = I_G \vec{\alpha} + \vec{r}_{C/P} \times m\vec{a}_C$$

Kinetic Energy

Particle: $T = \frac{1}{2}mv^2$ Rigid Body: $T = \frac{1}{2}mv_C^2 + \frac{1}{2}I_C\omega^2$

Work

Force $U_{1-2} = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s} = \int_{s_1}^{s_2} F_t ds$

Linear Spring Force $U_{1-2} = -\frac{1}{2}k(\delta_2^2 - \delta_1^2)$

Couple, Moment $U_{1-2} = \int_{\theta_1}^{\theta_2} M d\theta$

Potential Energy

$$V = V_g + V_e, \quad \text{where } V_g = \pm Wh$$

$$V_e = +\frac{1}{2}k\delta^2 + \frac{1}{2}k_\theta\theta^2$$

Principle of Work and Energy

$$T_1 + U_{1-2} = T_2$$

$$T_1 + V_{g1} + V_{e1} + U_{1-2} = T_2 + V_{g2} + V_{e2}$$

$$\text{or } U_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

Conservation of Energy

$U_{1-2} = 0$ Therefore $\Delta T + \Delta V = 0$
 or $T_1 + V_1 = T_2 + V_2$

Power and Efficiency

$$P = \frac{dU}{dt} = \vec{F} \cdot \vec{v} + \vec{M} \cdot \vec{\omega}$$

$$\eta_{mech} = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$$

Principle of Impulse - Momentum

$$(\text{Momentum})_1 + (\text{Impulse})_{1-2} = (\text{Momentum})_2$$

Linear Impulse and Momentum

Particle: $m\vec{v}_1 + \sum \int \vec{F} dt = m\vec{v}_2$

Rigid Body: $m(\vec{v}_C)_1 + \sum \int \vec{F} dt = m(\vec{v}_C)_2$

Conservation of Linear Momentum

$$\sum (m\vec{v})_1 = \sum (m\vec{v})_2$$

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Coefficient of Restitution

$$e = \frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}}$$

$$= - \frac{\text{normal component of relative velocity after impact}}{\text{normal component of relative velocity before impact}}$$

Angular Impulse and Momentum

Particle/Rigid Body: $(\vec{H}_O)_1 + \sum \int \vec{M}_O dt = (\vec{H}_O)_2$

for particle

$$\vec{H}_O = (mv)(d) = \vec{r}_{P/O} \times (m\vec{v})$$

for rigid body

$$\vec{H}_O = I_G \vec{\omega} + (mv_G)(d) = I_G \vec{\omega} + \vec{r}_{G/O} \times (m\vec{v}_G)$$

Rigid Body about G:

$$(\vec{H}_G)_1 + \sum \int \vec{M}_G dt = (\vec{H}_G)_2$$

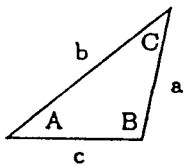
where $\vec{H}_G = I_G \vec{\omega}$

Conservation of Angular Momentum

$$\sum (\vec{H}_O \text{ or } \vec{c})_1 = \sum (\vec{H}_O \text{ or } \vec{c})_2$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$



FPS Units

- 1 inch = 2.54 cm (exact)
- 12 inches = 1 foot = 0.3048 metres (exact)
- 3 feet = 1 yard
- 5280 feet = 1 mile = 1.6093 kilometres
- 6076 feet = 1 nautical mile = 1.852 kilometres (exact)
- 1 knot = 1.852 km/hr
- $g = 32.2 \text{ ft/s}^2 = 9.81 \text{ m/s}^2$
- 1 slug weighs 32.2 pounds
- 1 kg weighs 9.81 Newtons
- 1 lb = 4.4482 N
- 1 slug = 14.5938 kg