École de gestion TELFER School of Management

LIÉE au LINKED with LEADERSHIP

ADM 2304 – Final Examination

APPLIED STATISTICAL METHODS IN BUSINESS

Winter 2014

PRINT YOUR NAME: ____

STUDENT NUMBER: ________________________________

SECTION (circle one): M N P Q R S

Exam: **9 pages (including cover page)**

Appendices: **9 pages of Minitab output and 4 pages of statistical tables**

Complete all tests with hypotheses, test statistic, critical value, decision, and conclusion. Use the 0.05 significance level unless otherwise indicated. Explain your answers if asked.

Calculators and one sheet of notes (8.5 x 14 in.) are allowed.

Please hand back all exam materials (exam booklet, appendices and tables), but keep your personal sheet of notes for future reference.

Question	**Value**	**Mark**
1	6	
2	8	
3	4	
4	10	
5	15	
6	22	
Total	**65**	

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ____

Question 1. [6 marks]

A shortage of long term care capacity has long hindered our health care system from providing appropriate care to Canadian citizens. One issue is whether the age of entry into long term care is related to the living arrangements of residents before entry. The following data were compiled:

	<60	60-69	70-79	80-89	90-99	>100	Totals
Alone	14	48	140	524	278	12	1016
Non-Private Residence	16	28	98	285	192	8	627
Spouse/Life Partner	22	99	374	783	232	4	1514
With Family	37	32	127	311	209	10	726
Totals							

The data outlines four potential pre-admission living arrangements – alone, non-private residence, living with spouse and living with family. The columns represents the age of residents at time of admission. Using the partially completed output in the Appendix A, perform the appropriate hypothesis test to determine if there is indeed a relationship between the age of entry and the living arrangement prior to admission. Use a 5% significance level. (You should show the calculation of the missing chi-square contribution.)

<u>Question 2.</u> [8 marks]

(a) Using the same data, test whether the proportions of residents entering long term care before the age of 70 are different for those living alone versus those living with family. Use a 5% significance level and the critical value approach.

[4]

(b) Calculate a 95% confidence interval for the difference between the proportion of residents entering long term care before the age of 70 among those living alone and the proportion of residents entering before the age of 70 among those living with family. Does this confirm your conclusion in part (a)?

[3]

(c) What key assumption justifies the use of the Z distribution in this question?

[1]

Question 3. [4 marks]

Another issue of whether appropriate care can be provided is whether there has been a shift in the age distribution of incoming long term care residents. Some changes (such as increased severe obesity and increased dementia) might lead to earlier entry into long term care while increased life expectancy might mean that people require long term care later. In 2008, the **population** of residents entering long term care had the following distribution:

	<60	60-69	70-79	80-89	90-99	>100
2008	2.96%	5.45%	18.21%	48.71%	23.57%	1.10%

In 2012, a random sample of residents entering long term care were distributed as follows:

	<60	60-69	70-79	80-89	90-99	>100	Total
2012	33	59	234	649	383	17	1375

Perform the appropriate hypothesis test to determine whether there has been an overall shift in the distribution of the age of residents at time of admission into long term care over the 4 year period. Use a 1% significance level. Your test must consider all the age groups.

Question 4. [10 marks]

Appendix B shows the average salaries for male and female assistant professors in the Faculty of Arts at 22 different universities. Use the 0.05 level of significance in all tests.

(a) Explain whether the two samples are independent or paired.

[1]

(b) With specific reference to the appropriate boxplot(s), explain whether a parametric or non-parametric test is more appropriate to test for a difference in average salaries (mean or median).

[2]

(c) Without regard to your answer in (b), perform the appropriate non-parametric test.

[3]

(d) Without regard to your answer in (b), perform the appropriate t-test.

[3]

(e) Explain how the paired t-test results are equivalent to the two-way analysis of variance results.

[1]

Question 5. [15 marks]

A consumer organization wanted to examine the effect of age and gender on the amount (in hundreds of dollars) used car dealerships were willing to pay for a used car. Twelve “owners” (six male and six female) were recruited from each of three age groups (young, middle and senior). Each “owner” was randomly assigned to one of thirty-six used car dealers and brought the same car to solicit a cash offer. The data and analyses can be found in Appendix C.

(a) Examine the residual plot and explain whether the model assumptions are justified.

[2]

(b) Describe the effects of age and gender on the average offer, using the interaction plot. Explain whether age and gender interact with each other in how they affect the offer.

[3]

(c) Test whether the effect of age depends on gender. Use the 5% level of significance.

[3]

(d) Now test the effect of age on the average offer. Use the 5% level of significance.

[3]

(e) Using a Bonferroni family confidence level of 95%, can you conclude that middle age “owners” receive higher cash offers than either of the other two age groups?

[4]

Question 6. **[22 marks]**

To model the Canadian/U.S. dollar exchange rate ('ExchangeRt'), stock indices for various sectors of the economy are used as predictors. Some of the variables used in the four models shown in Appendix D are:

- ExchangeRt: The Canadian dollar equivalent to the U.S. dollar.
- Energy: Index of energy stocks, including oil and gas.
- Financial: Index of Banks and other financial institutions
- Industrial: Industrial Index
- Materials: Index of Lumber, Potash and other materials
- M&M: Index for Metals and Mining sector
- Utilities: Index for the power production sector
- IT: Information technology sector index

The following questions pertain to **Model 1** [4 marks]

(a) What problem(s) can you identify in the residual plots?
[1]

(b) What is the simple linear regression equation which relates 'ExchangeRt' to the predictor variable 'Energy'?
[1]

(c) Explain why the negative coefficient of the 'Energy' index makes economic sense.
[1]

(d) What is the correlation coefficient between 'ExchangeRt' and 'Energy'?
[1]

Comparison of models [3 marks]

(e) Between Model 2 and Model 3, which is the better model? Explain briefly.
[2]

(f) The high Variance Inflation Factors suggest that there may be a problem with multicollinearity. Describe how this might affect the regression coefficients.
[1]

Answer the following questions using **Model 4** [15 marks]

(g) Explain if the linear model assumptions are justified.
[2]

(h) How might you improve the fit of Model 4? (Only one suggestion will be marked.)
[1]

(i) Test whether this model is useful. Use the 5% level of significance.
[3]

(j) Test at the 1% level of significance whether the ‘Financials’ index is important to the model. State your conclusion carefully.

[3]

(k) Explain the meaning of the estimated coefficient of the ‘Financials’ index? Be as precise as possible.

[2]

(l) Using Model 4, calculate a 99% interval prediction for ‘ExchangeRt’ when the 'Financials' index is 190 and the 'Materials' index is 400.

[2]

(m) If ‘ExchangeRt’ is regressed on the residuals from Model 4, the R-square is 0.0416. How is this related to the R-square of Model 4? Explain the relationship.

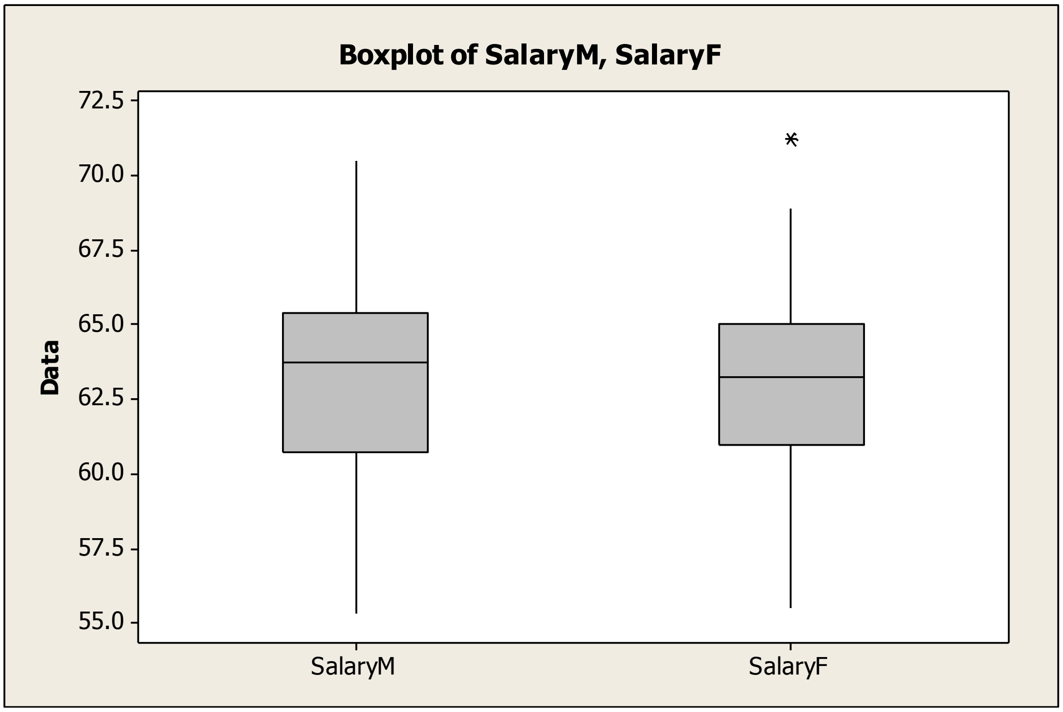
[2]

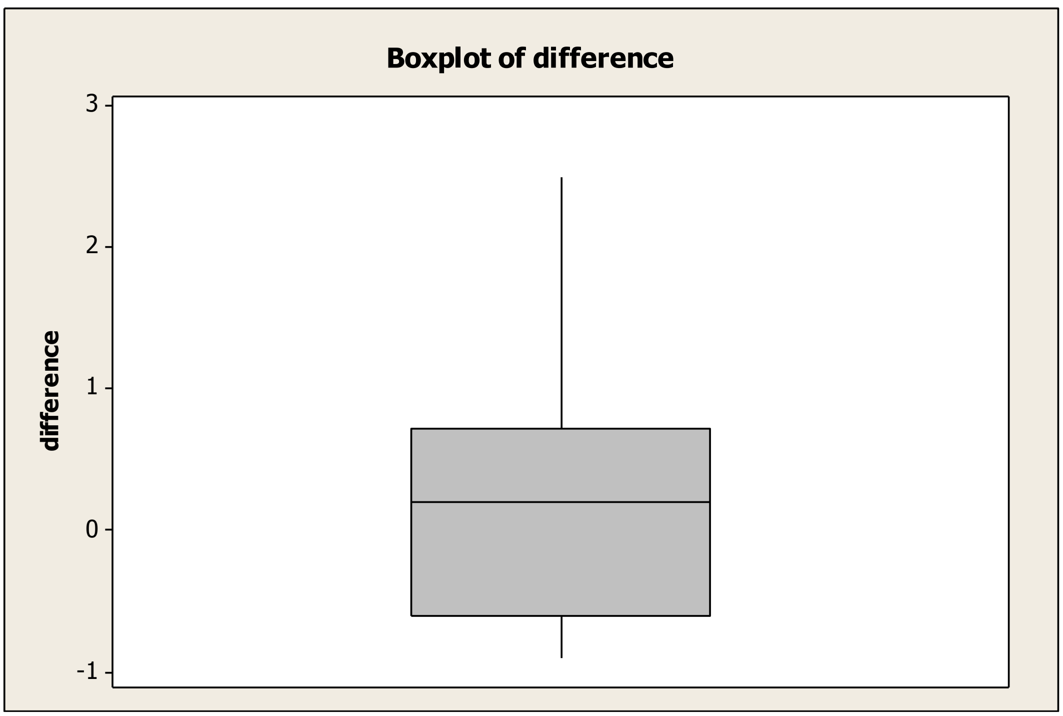
Appendix A

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

           <60   60-69   70-79   80-89   90-99   >100  Total
    1       14      48     140     524     278     12   1016
         23.29   54.16  193.36  497.93  238.37   8.90
         3.704   0.701  14.726   1.365   6.590  1.083

    2       16      28      98     285     192      8    627
         14.37   33.42  119.33  307.28  147.10   5.49
         0.185   0.880   3.812   1.616  13.704  1.147

    3       22      99     374     783     232      4   1514
         34.70   80.71  288.14  741.99  355.20  13.26
         4.649   4.145  25.585   2.267  42.733  6.464

    4       37      32     127     311     209     10    726
         _____   38.70  138.17  355.80  170.33   6.36
        ______   1.161   0.903   5.641   8.780  2.088

Total       __     207     739    1903     911     34   ____
```

Appendix B

University	SalaryM	SalaryF	difference
1	64.5	63.9	0.6
2	60.5	61.2	-0.7
3	65.1	65	0.1
4	65.7	64.2	1.5
5	61.5	62.4	-0.9
6	64.4	64.1	0.3
7	62.1	62.7	-0.6
8	60.7	59.9	0.8
9	63.7	61.2	2.5
10	65.3	65.5	-0.2
11	60.7	60.2	0.5
12	64.2	64.8	-0.6
13	69.6	68.7	0.9
14	60.5	60	0.5
15	63.8	63.8	0
16	61.7	62.4	-0.7
17	62.8	61.7	1.1
18	68.5	68.9	-0.4
19	70.5	71.2	-0.7
20	55.3	55.5	-0.2
21	58.6	58	0.6
22	65.8	65.1	0.7





Two-Sample T-Test and CI: SalaryM, SalaryF

```
Two-sample T for SalaryM vs SalaryF

          N   Mean  StDev  SE Mean
SalaryM  22  63.43   3.55     0.76
SalaryF  22  63.20   3.60     0.77
```

```
Difference = mu (SalaryM) - mu (SalaryF)
Estimate for difference:  0.231818
95% CI for difference:  (_________, ________)
T-Test of difference = 0 (vs not =): T-Value = ____  P-Value = _____  DF = __
Both use Pooled StDev = 3.5738
```

Paired T-Test and CI: SalaryM, SalaryF

```
Paired T for SalaryM - SalaryF

             N      Mean     StDev   SE Mean
SalaryM     22   63.4318    3.5462    0.7561
SalaryF     22   63.2000    3.6012    0.7678
Difference  22  0.231818  0.845935  0.180354


95% CI for mean difference: (_________, ________)
T-Test of mean difference = 0 (vs not = 0): T-Value = ____  P-Value = _____
```

Two-way ANOVA: Salary versus University, Gender1

```
Source      DF       SS       MS      F      P
University  21  528.914  25.1864  70.39  0.000
Gender1      1    0.591   0.5911   1.65  _____
Error       21    7.514   0.3578
Total       43  537.019

S = 0.5982   R-Sq = 98.60%   R-Sq(adj) = 97.14%


                  Individual 95% CIs For Mean Based on
                  Pooled StDev
Gender1    Mean   ---+---------+---------+---------+------
Female  63.2000   (------------*------------)
Male    63.4318              (-------------*------------)
                  ---+---------+---------+---------+------
                  63.00     63.20     63.40     63.60
```

Wilcoxon Signed Rank Test: difference

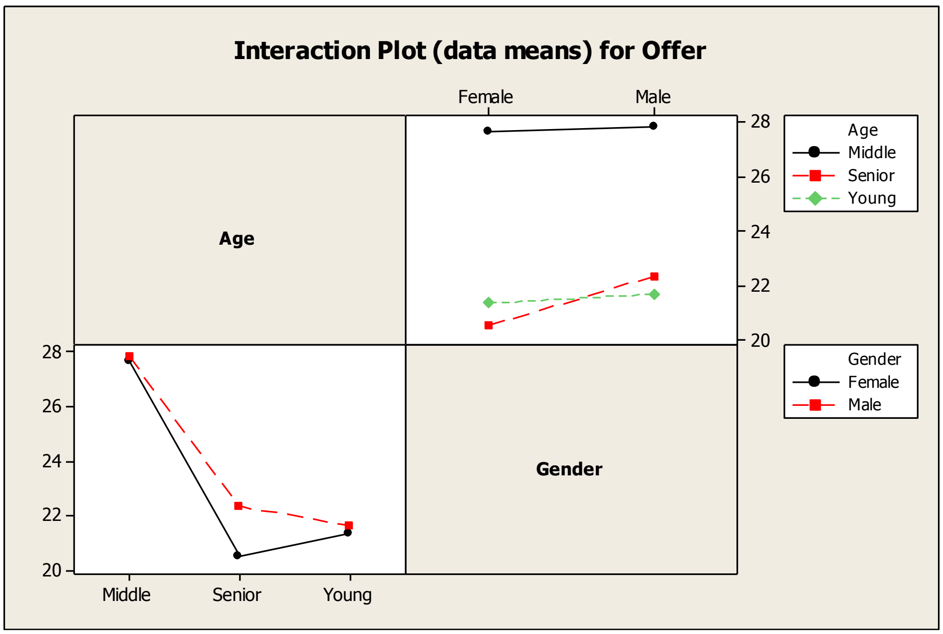
```
Test of median = 0.000000 versus median not = 0.000000

                   N
                 for  Wilcoxon          Estimated
             N  Test  Statistic      P     Median
difference  22    21      143.5  0.339     0.1500
```

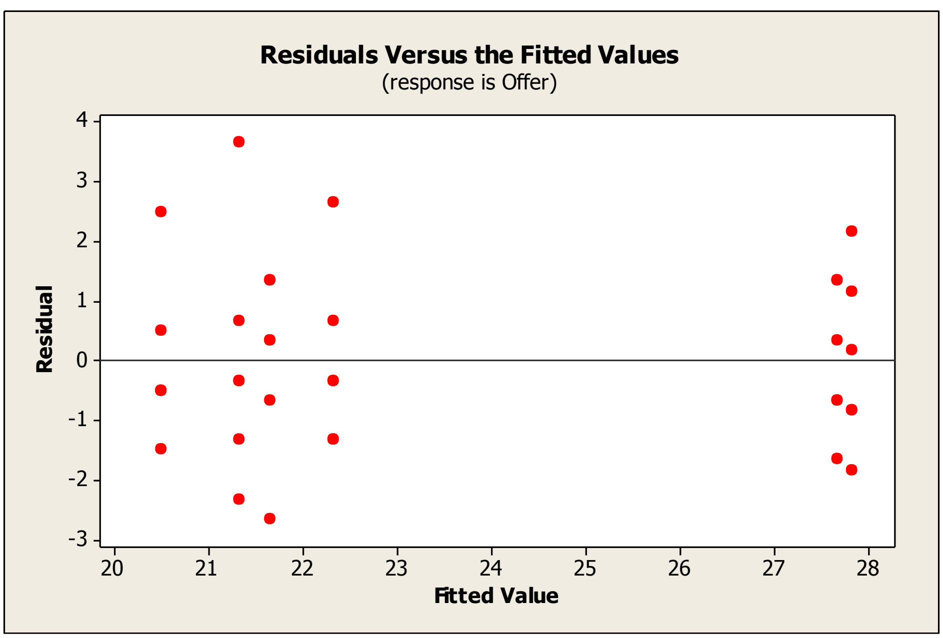
Mann-Whitney Test and CI: SalaryM, SalaryF

```
          N  Median
SalaryM  22  63.750
SalaryF  22  63.250

Point estimate for ETA1-ETA2 is 0.300
95.0 Percent CI for ETA1-ETA2 is (-1.700,2.400)
W = 510.0
Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.7336
The test is significant at 0.7335 (adjusted for ties)
```

Appendix C

Age	Male			Female		
Young	21	23	19	21	22	20
	22	22	23	21	19	25
Middle	30	29	26	26	29	27
	28	27	27	28	27	29
Senior	25	22	23	23	19	20
	21	22	21	21	20	20





Two-way ANOVA: Offer versus Age, Gender

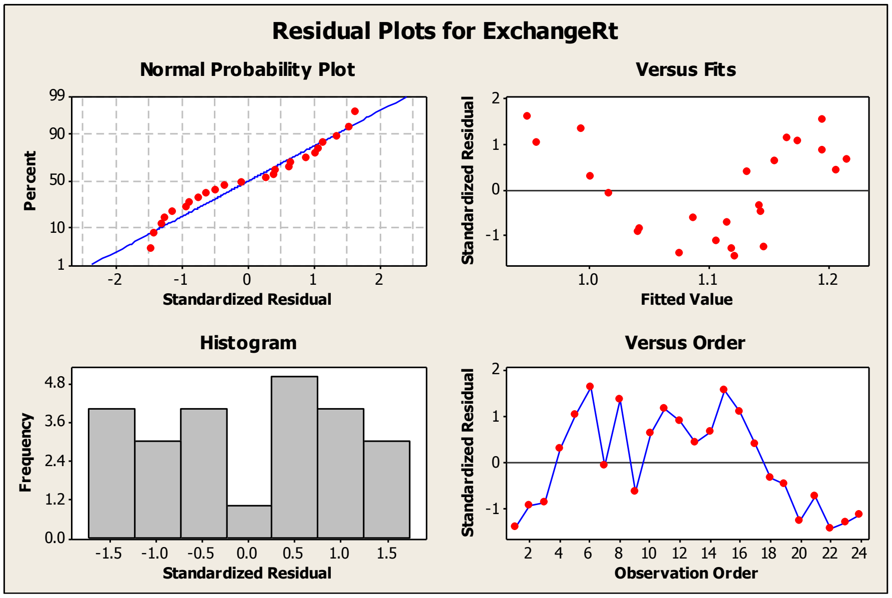
```
Source       DF       SS       MS      F      P
Age           2  316.722  158.361  66.29  _____
Gender        1    5.444    5.444   2.28  _____
Interaction   2    5.056    2.528   1.06  _____
Error        30   71.667    2.389
Total        35  398.889

S = 1.546   R-Sq = 82.03%   R-Sq(adj) = 79.04%


                   Individual 95% CIs For Mean Based on
                   Pooled StDev
Age        Mean    --------+---------+---------+---------+-
Middle  27.7500                             (---*---)
Senior  21.4167    (---*--)
Young   21.5000    (---*---)
                   --------+---------+---------+---------+-
                         22.5      25.0      27.5      30.0


                   Individual 95% CIs For Mean Based on
                   Pooled StDev
Gender     Mean    ------+---------+---------+---------+---
Female  23.1667    (-----------*------------)
Male    23.9444                 (-----------*-----------)
                   ------+---------+---------+---------+---
                       22.80     23.40     24.00     24.60
```

Appendix D

Model 1

Regression Analysis: ExchangeRt versus Energy

```
The regression equation is
ExchangeRt = 1.43 - 0.00109 Energy

Predictor        Coef    SE Coef      T      P
Constant      1.42989    0.04184  34.17  0.000
Energy     -0.0010857  0.0001358  _____  _____

S = 0.0472508   R-Sq = _____   R-Sq(adj) = 73.2%

Analysis of Variance

Source          DF       SS       MS      F      P
Regression       1  0.14275  0.14275  _____  _____
Residual Error  22  0.04912  0.00223
Total           23  0.19187
```



Model 2

Regression Analysis: ExchangeRt versus Financials, Industrials, ...

```
The regression equation is
ExchangeRt = 1.68 - 0.00200 Financials + 0.00174 Industrials
             - 0.000337 Materials - 0.000077 M&M - 0.00132 Utilities
```

Predictor	Coef	SE Coef	T	P	VIF
Constant	1.67814	0.04162	40.32	0.000	
Financials	-0.0019995	0.0004481	-4.46	0.000	10.135
Industrials	0.001739	0.001199	1.45	0.164	25.996
Materials	-0.0003365	0.0001866	-1.80	0.088	8.325
M&M	-0.00007700	0.00004241	-1.82	0.086	7.617
Utilities	-0.0013209	0.0006196	-2.13	0.047	17.081

S = 0.0184611   R-Sq = 96.8%   R-Sq(adj) = 95.9%

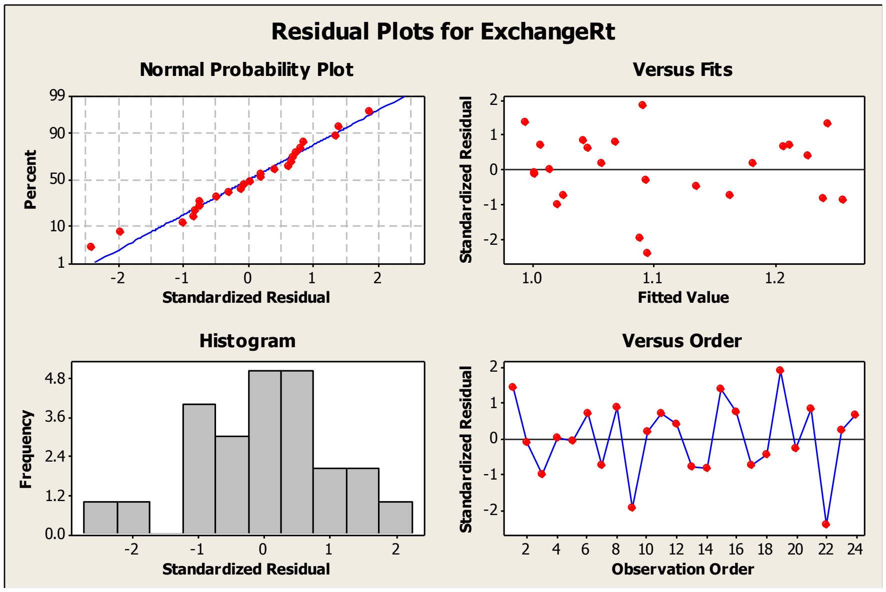
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	5	0.185735	0.037147	109.00	0.000
Residual Error	18	0.006135	0.000341		
Total	23	0.191869			

Unusual Observations

Obs	Financials	ExchangeRt	Fit	SE Fit	Residual	St Resid
22	166	1.05485	1.09439	0.00857	-0.03954	-2.42R

R denotes an observation with a large standardized residual.



Model 3

Regression Analysis: ExchangeRt versus Financials, Industrials, ...

```
The regression equation is
ExchangeRt = 1.68 - 0.00202 Financials + 0.00131 Industrials
             - 0.000398 Materials - 0.000083 M&M - 0.00132 Utilities
             + 0.00223 IT
```

Predictor	Coef	SE Coef	T	P	VIF
Constant	1.68310	0.04215	39.93	0.000	
Financials	-0.0020195	0.0004506	-4.48	0.000	10.158
Industrials	0.001310	0.001291	1.01	0.325	29.911
Materials	-0.0003983	0.0001991	-2.00	0.062	9.395
M&M	-0.00008297	0.00004309	-1.93	0.071	7.794
Utilities	-0.0013223	0.0006223	-2.12	0.049	17.081
IT	0.002230	0.002426	0.92	0.371	12.485

S = 0.0185411   R-Sq = 97.0%   R-Sq(adj) = 95.9%

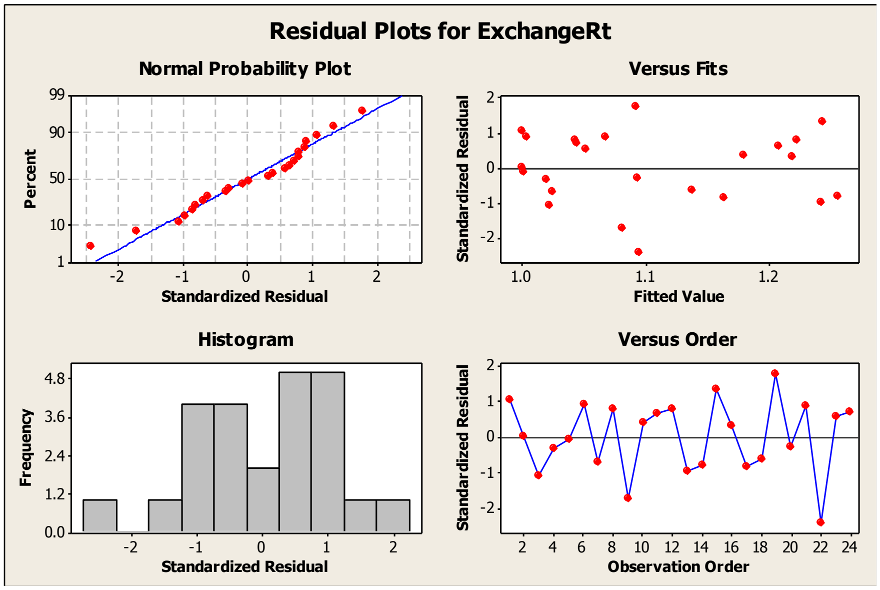
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	6	0.186025	0.031004	90.19	0.000
Residual Error	17	0.005844	0.000344		
Total	23	0.191869			

Unusual Observations

Obs	Financials	ExchangeRt	Fit	SE Fit	Residual	St Resid
22	166	1.05485	1.09486	0.00862	-0.04001	-2.44R

R denotes an observation with a large standardized residual.

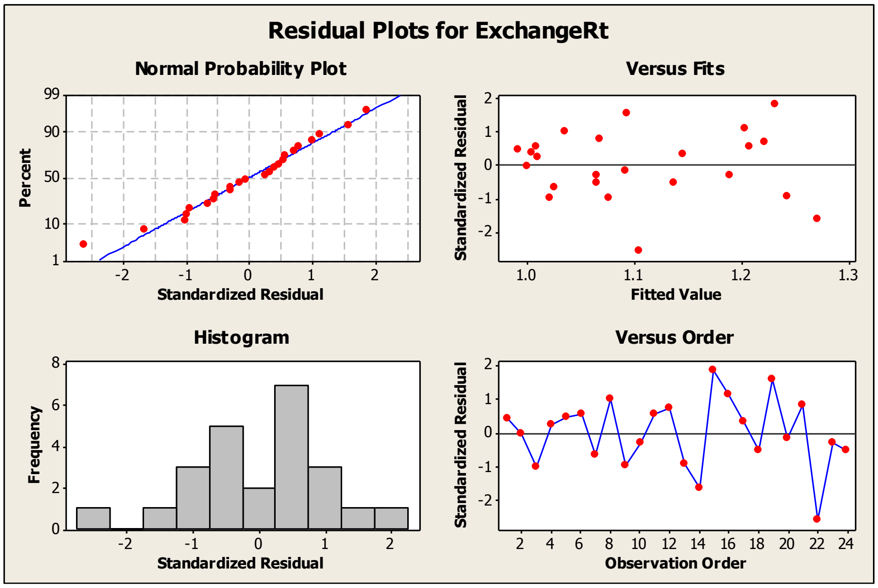


Model 4

Regression Analysis: ExchangeRt versus Financials, Materials

```
The regression equation is
ExchangeRt = 1.65 - 0.00226 Financials - 0.000559 Materials

Predictor         Coef    SE Coef       T      P    VIF
Constant       1.64602    0.02504   65.74  0.000
Financials  -0.0022571  0.0002247  ______  _____  2.285
Materials   -0.0005589  0.0001032   -5.41  0.000  2.285

S = 0.0194924   R-Sq = 95.8%   R-Sq(adj) = 95.4%

Analysis of Variance

Source          DF        SS        MS       F      P
Regression       2  0.183890  0.091945  ______  _____
Residual Error  21  0.007979  0.000380
Total           23  0.191869

Source      DF    Seq SS
Financials   1  0.172755
Materials    1  0.011135

Unusual Observations

Obs  Financials  ExchangeRt      Fit   SE Fit  Residual  St Resid
 22         166     1.05485  1.10422  0.00411  -0.04937     -2.59R

Predicted Values for New Observations

New Obs      Fit   SE Fit         99% CI                99% PI
      1  0.99361  0.00768  (_______, _______)   (_______, _______)

Values of Predictors for New Observations

New Obs  Financials  Materials
      1         190        400
```

Residual Plots for ExchangeRt



Best Subsets Regression: ExchangeRt versus Energy, Financials, ...

Response is ExchangeRt

Vars	R-Sq	R-Sq(adj)	Mallows Cp	S	Energy	Financials	Industrials	Materials	M&M	Utilities	IT
1	90.0	89.6	32.8	0.029476		X					
1	82.0	81.2	75.6	0.039653			X				
2	95.8	95.4	4.1	0.019492		X		X			
2	93.1	92.4	18.8	0.025195					X	X	
3	96.0	95.4	5.2	0.019584		X			X	X	
3	96.0	95.4	5.3	0.019618		X		X	X		
4	96.4	95.7	4.9	0.018991		X		X	X	X	
4	96.2	95.4	6.0	0.019524		X	X		X	X	
5	96.8	95.9	5.0	0.018461		X	X	X	X	X	
5	96.8	95.9	5.1	0.018556		X		X	X	X	X
6	97.0	95.9	6.2	0.018541		X	X	X	X	X	X
6	96.8	95.7	6.8	0.018905	X	X	X	X	X	X	
7	97.0	95.7	8.0	0.019021	X	X	X	X	X	X	X

Standard Normal Distribution

P(Z < z) **(z negative)**

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) (z positive)

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

t_α

α = P(t > t_α) = one-tail probability

df	0.100	0.050	0.025	0.010	0.009	0.008	0.005	0.001	0.0005	0.0004	0.0001
1	3.08	6.31	12.71	31.82	35.36	39.78	63.66	318.31	636.62	837.66	3183.10
2	1.89	2.92	4.30	6.96	7.35	7.81	9.92	22.33	31.60	36.25	70.70
3	1.64	2.35	3.18	4.54	4.72	4.93	5.84	10.21	12.92	14.18	22.20
4	1.53	2.13	2.78	3.75	3.87	4.01	4.60	7.17	8.61	9.25	13.03
5	1.48	2.02	2.57	3.36	3.46	3.57	4.03	5.89	6.87	7.29	9.68
6	1.44	1.94	2.45	3.14	3.23	3.32	3.71	5.21	5.96	6.28	8.02
7	1.41	1.89	2.36	3.00	3.07	3.16	3.50	4.79	5.41	5.67	7.06
8	1.40	1.86	2.31	2.90	2.97	3.04	3.36	4.50	5.04	5.26	6.44
9	1.38	1.83	2.26	2.82	2.89	2.96	3.25	4.30	4.78	4.98	6.01
10	1.37	1.81	2.23	2.76	2.83	2.89	3.17	4.14	4.59	4.77	5.69
11	1.36	1.80	2.20	2.72	2.78	2.84	3.11	4.02	4.44	4.60	5.45
12	1.36	1.78	2.18	2.68	2.74	2.80	3.05	3.93	4.32	4.47	5.26
13	1.35	1.77	2.16	2.65	2.71	2.77	3.01	3.85	4.22	4.37	5.11
14	1.35	1.76	2.14	2.62	2.68	2.74	2.98	3.79	4.14	4.28	4.99
15	1.34	1.75	2.13	2.60	2.66	2.71	2.95	3.73	4.07	4.21	4.88
16	1.34	1.75	2.12	2.58	2.64	2.69	2.92	3.69	4.01	4.15	4.79
17	1.33	1.74	2.11	2.57	2.62	2.67	2.90	3.65	3.97	4.09	4.71
18	1.33	1.73	2.10	2.55	2.60	2.66	2.88	3.61	3.92	4.04	4.65
19	1.33	1.73	2.09	2.54	2.59	2.64	2.86	3.58	3.88	4.00	4.59
20	1.33	1.72	2.09	2.53	2.58	2.63	2.85	3.55	3.85	3.97	4.54
21	1.32	1.72	2.08	2.52	2.57	2.62	2.83	3.53	3.82	3.93	4.49
22	1.32	1.72	2.07	2.51	2.56	2.61	2.82	3.50	3.79	3.91	4.45
23	1.32	1.71	2.07	2.50	2.55	2.60	2.81	3.48	3.77	3.88	4.42
24	1.32	1.71	2.06	2.49	2.54	2.59	2.80	3.47	3.75	3.85	4.38
25	1.32	1.71	2.06	2.49	2.53	2.58	2.79	3.45	3.73	3.83	4.35
26	1.31	1.71	2.06	2.48	2.53	2.58	2.78	3.43	3.71	3.81	4.32
27	1.31	1.70	2.05	2.47	2.52	2.57	2.77	3.42	3.69	3.79	4.30
28	1.31	1.70	2.05	2.47	2.51	2.56	2.76	3.41	3.67	3.78	4.28
29	1.31	1.70	2.05	2.46	2.51	2.56	2.76	3.40	3.66	3.76	4.25
30	1.31	1.70	2.04	2.46	2.50	2.55	2.75	3.39	3.65	3.75	4.23
31	1.31	1.70	2.04	2.45	2.50	2.55	2.74	3.37	3.63	3.73	4.22
32	1.31	1.69	2.04	2.45	2.49	2.54	2.74	3.37	3.62	3.72	4.20
33	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.36	3.61	3.71	4.18
34	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.35	3.60	3.70	4.17
35	1.31	1.69	2.03	2.44	2.48	2.53	2.72	3.34	3.59	3.69	4.15
36	1.31	1.69	2.03	2.43	2.48	2.53	2.72	3.33	3.58	3.68	4.14
37	1.30	1.69	2.03	2.43	2.48	2.52	2.72	3.33	3.57	3.67	4.13
38	1.30	1.69	2.02	2.43	2.47	2.52	2.71	3.32	3.57	3.66	4.12
39	1.30	1.68	2.02	2.43	2.47	2.52	2.71	3.31	3.56	3.65	4.10
40	1.30	1.68	2.02	2.42	2.47	2.52	2.70	3.31	3.55	3.65	4.09
50	1.30	1.68	2.01	2.40	2.45	2.49	2.68	3.26	3.50	3.59	4.01
60	1.30	1.67	2.00	2.39	2.43	2.48	2.66	3.23	3.46	3.55	3.96
100	1.29	1.66	1.98	2.36	2.41	2.45	2.63	3.17	3.39	3.47	3.86
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.58	3.10	3.30	3.38	3.73

Treat df = 1000 as infinity

Chi-square distribution

$\chi 2_{\alpha;\nu}$

$\alpha = P(\chi^2 \geq \chi^2_{\alpha;\nu})$

ν	0.995	0.990	0.980	0.975	0.950	0.900	0.100	0.050	0.025	0.020	0.010	0.005
1	0.00	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	5.41	6.63	7.88
2	0.01	0.02	0.04	0.05	0.10	0.21	4.61	5.99	7.38	7.82	9.21	10.60
3	0.07	0.11	0.18	0.22	0.35	0.58	6.25	7.81	9.35	9.84	11.34	12.84
4	0.21	0.30	0.43	0.48	0.71	1.06	7.78	9.49	11.14	11.67	13.28	14.86
5	0.41	0.55	0.75	0.83	1.15	1.61	9.24	11.07	12.83	13.39	15.09	16.75
6	0.68	0.87	1.13	1.24	1.64	2.20	10.64	12.59	14.45	15.03	16.81	18.55
7	0.99	1.24	1.56	1.69	2.17	2.83	12.02	14.07	16.01	16.62	18.48	20.28
8	1.34	1.65	2.03	2.18	2.73	3.49	13.36	15.51	17.53	18.17	20.09	21.95
9	1.73	2.09	2.53	2.70	3.33	4.17	14.68	16.92	19.02	19.68	21.67	23.59
10	2.16	2.56	3.06	3.25	3.94	4.87	15.99	18.31	20.48	21.16	23.21	25.19
11	2.60	3.05	3.61	3.82	4.57	5.58	17.28	19.68	21.92	22.62	24.72	26.76
12	3.07	3.57	4.18	4.40	5.23	6.30	18.55	21.03	23.34	24.05	26.22	28.30
13	3.57	4.11	4.77	5.01	5.89	7.04	19.81	22.36	24.74	25.47	27.69	29.82
14	4.07	4.66	5.37	5.63	6.57	7.79	21.06	23.68	26.12	26.87	29.14	31.32
15	4.60	5.23	5.98	6.26	7.26	8.55	22.31	25.00	27.49	28.26	30.58	32.80
16	5.14	5.81	6.61	6.91	7.96	9.31	23.54	26.30	28.85	29.63	32.00	34.27
17	5.70	6.41	7.26	7.56	8.67	10.09	24.77	27.59	30.19	31.00	33.41	35.72
18	6.26	7.01	7.91	8.23	9.39	10.86	25.99	28.87	31.53	32.35	34.81	37.16
19	6.84	7.63	8.57	8.91	10.12	11.65	27.20	30.14	32.85	33.69	36.19	38.58
20	7.43	8.26	9.24	9.59	10.85	12.44	28.41	31.41	34.17	35.02	37.57	40.00

Fisher's F distribution

$F_{\alpha;\nu 1;\nu 2}$

$\alpha = P(F \geq f_{\alpha;\nu 1;\nu 2}) =$ 0.050

ν_2	1	2	3	4	5	6	7	8	9	10	11	12
10	5.0	4.1	3.7	3.5	3.3	3.2	3.1	3.1	3.0	3.0	2.9	2.9
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.93	1.89
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.91	1.88
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.90	1.86
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80