École de gestion TELFER School of Management

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ADM 2304 – Final Examination
APPLIED STATISTICAL METHODS IN BUSINESS
Winter 2011

PRINT YOUR NAME: ______________________________

STUDENT NUMBER: ______________________

SECTION (please circle one): M N P Q R

Time allowed: **3 hours**
Question booklet: **10 pages (including front cover)**
Appendices: **7 pages of Minitab output and 5 pages of statistical tables**

Instructions:

1. ***Complete all tests with hypotheses, test statistic, critical value, decision, and conclusion. Use the .05 level of significance unless otherwise indicated.***
2. Calculators and one sheet of notes (8.5 x 14 in.) are allowed.
3. You must hand back all exam materials (exam question booklet, appendices and tables), but please keep your personal sheet of notes for future use.
4. Have a great summer.

Question	Value	Mark
1	17	
2	13	
3	15	
4	25	
Total	**70**	

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ______________________________

Question 1. [17 marks]

At the risk of encouraging poor incentives for attending school, researchers sought to determine whether a university business degree or higher leads to higher salaries. They selected a random sample of 59 executives from companies with assets over $1 million and kept track of their educational backgrounds. The salaries of the 15 high-school graduates had a sample mean of $49.07 (in thousands) and a sample standard deviation of $13.16; the remaining executives with at least an undergraduate degree had a sample mean of $75.55 and a sample standard deviation of $15.09.

a. Identify the appropriate parametric test and comment on whether the assumptions required are warranted based on the boxplots in Appendix A.

[2]

b. Identify the appropriate non-parametric test and explain why it would be a valid test even if the assumptions in (a) were not warranted.

[2]

c. Perform the parametric test you chose in (a) to answer the hypothesis posed at the beginning of the question. Use a significance level of 0.05. State any additional assumptions that are reasonable and necessary to make.

[5]

Assumption(s):

d. The researchers belatedly realized that they would like to analyze the data to determine whether having a graduate degree further enhances the chances of earning a higher salary. They divided their sample of undergraduate degree executives into those who had only an undergraduate degree and those who had a graduate degree as well. The sample mean for the 24 executives who only had an undergraduate degree (the rest had a graduate degree) was 72.71 and the sample standard deviation was 14.95 while for those who also had a graduate degree, the sample mean was 78.95 and the sample standard deviation was 14.91. They performed the appropriate analysis on all three sub-samples. Your task is to fill in the rest of the ANOVA table, but you should show also how the MSE = 211 might be calculated, using only the summary statistics in the following table:

[4] Start by summarizing the three samples:

Highest level of education	*High school only*	*Undergraduate degree*	*Graduate degree*
Mean	49.07	72.71	78.95
Standard deviation			
Sample size	15	24	20

MSE = 211 is calculated by the following numerical expression:

	Degrees of freedom	Sum of Squares	Mean Square	F
(Education) Factor	2	8268		
Error			211	
Total				

e. Using the results above, calculate a 95% Bonferroni confidence interval to determine whether there is a statistically significant difference between those with graduate degrees and those with only an undergraduate degree. To help you out, the necessary t critical value is 2.47, but you must explain how this was derived.

[4]

T = 2.47 is based on a tail probability of ________________ and __________ d.f.

Question 2. [13 marks]

A small independent stock broker has created four sector portfolios for her clients. Each portfolio always has five stocks that may change from year to year. The volatility of each stock is recorded for each year. Please refer to Appendix B.

a. Looking at the relevant graph, comment on the validity of the model assumptions.
[2]

b. Based on the partial analysis in Appendix B, perform the appropriate hypothesis test to determine if the impact of the type of portfolio on volatility varies depending on the year. Use a significance level of 0.05.
[4]

c. Are the tests for main effects relevant here? Why or why not?
[1]

d. Examine the interaction plots in Appendix B. Do they confirm or contradict your conclusion in (b)? Why is the hypothesis test still necessary when the interaction plots are available?

[2]

e. Use a Bonferroni confidence interval to determine whether the difference between the mean volatility of the Leisure portfolio in 2004 and the Retail portfolio in 2006 is statistically significantly non-zero. The required t-value is 3.597. How many pairwise comparisons are possible using the Bonferroni margin of error?

[4]

Number of possible pairwise comparisons = ________________________

Question 3. [15 marks]

Prior to the May 2, 2011 federal election, three political polls taken around April 14 gave the following Ontario results:

	Cons	Green	Lib	NDP	undecided	Total
Nanos	121	8	104	48	17	298
Forum	289	52	237	155	48	781
Ekos	195	45	185	70	23	518

A chi-square analysis yielded the following output:

Chi-Square Test: Cons, Green, Lib, NDP, undecided

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

         Cons  Green     Lib     NDP  undecided  Total
 Nanos    121      8     104      48         17    298
       ______   ____   98.15   50.94      16.42
       ______   ____   0.348   0.170      0.020

 Forum    289     52     237     155         48    781
       295.87  51.35  257.24  133.51      43.04
        0.160  0.008   1.592   3.460      0.573

 Ekos     195     45     185      70         23    518
       196.24   ____  170.61   88.55      28.54
        0.008   ____   1.213   3.886      1.077

Total     605    105     526     273         88   1597
```

a. Test at the .01 significance level whether there is evidence of overall differences in how the electorate is currently leaning (or not leaning) among the Ontario populations addressed by the three polling firms. Use a single test and a critical value approach.

[4]

b. What is the p-value of the result in part (a)? (Find the closest value possible on the table.) What observed count stands out as being the most responsible for the test statistic and p-value? Explain briefly.

[2]

c. The two largest polls (Forum and Ekos) show 32.3% and 37.4% Liberal support, respectively, among decided voters (n = 781-48 = 733 and n = 518-23 = 495). Test whether these values indicate a real difference between the two polls (at the .05 level)?

[4]

d. Seeing that the estimated Liberal support (32.3%) by the Forum is lower than that estimated by Ekos (37.4%), is it fair then to decide to perform the test above as a 1-sided test? Explain briefly.

[1]

e. During the 2008 federal election, 18.2% voted for the NDP in Ontario. During the weeks prior to the 2011 election, NDP support grew nationally. During the Easter weekend of 2011, a new poll of 1028 voters in Ontario showed 236 favouring the NDP. Test at the .05 significance level whether this constitutes evidence of a shift toward the NDP in Ontario.

[4]

Question 4. [25 marks]

A medical researcher wanted to establish a relationship between % body fat of male subjects, the response variable, and eight predictor variables:

Fat%:	% body fat
Weight:	body weight in pounds (lb.)
Height:	height in inches
Neck:	circumference of neck in cm
Chest:	circumference of chest in cm
Abdomen:	circumference of abdomen in cm
Hip:	circumference around hip in cm
Thigh:	circumference of thigh in cm
RegExer:	1 if regular physical exercise is done and '0' otherwise.

a. Carefully examine the three models and specify which model is the best one. Justify your choice on the basis of three numerical criteria. You will need to calculate some of these summary values.

[3]

b. Based on the output given, explain briefly if this best model satisfies the basic assumptions of the regression model.

[2]

Notwithstanding your answers above, please use **Model 2** for the remaining sub-questions.

c. Test whether the model is useful for predicting % body fat, using a 5% significance level.

[3]

d. What steps would you take to improve this model? Explain your rationale.
[2]

e. Explain if there are any specific problems with multicollinearity (refer to specific numerical evidence).
[2]

f. Based on the numerical values referred to in part (e), calculate the coefficient of determination (R^2 value) of this specific predictor variable when regressed against the other predictor variables. What does the R^2 value suggest?
[2]

g. Explain the meaning of the estimated coefficient of the 'RegExer' variable. Be as precise as possible.
[2]

h. Test whether regular exercise makes a difference to % body fat. State your conclusion as carefully as possible.

[3]

Suppose the weight is 170 lb, the neck is 38 cm, the abdomen is 90 cm, the thigh is 58 cm, and regular exercise is performed.

i. Calculate the point estimate/'fitted value' of 'Fat%'.

[1]

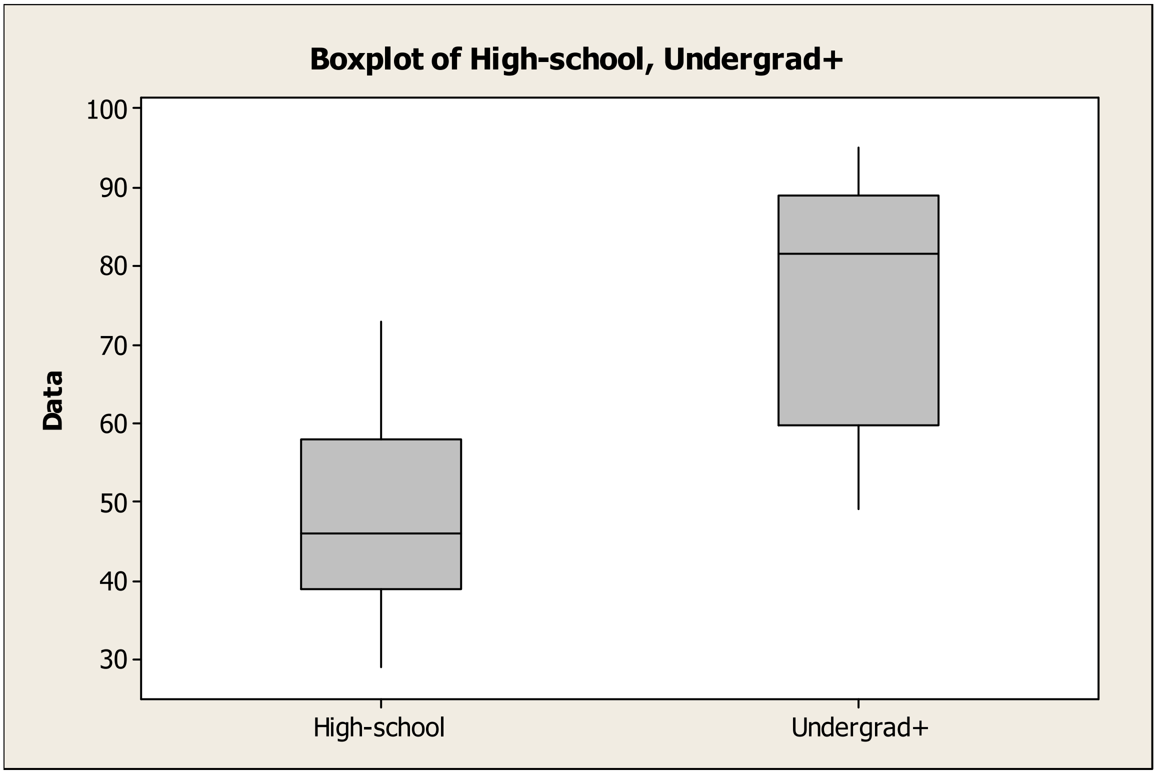
j. Now calculate a 95% confidence interval for % body fat, given the predictor variable values above. Explain what this interval estimates.

[3]

k. Modify the interval above to obtain a 95% prediction interval for the % body fat for a specific individual with the given characteristics.

[2]

Appendix A

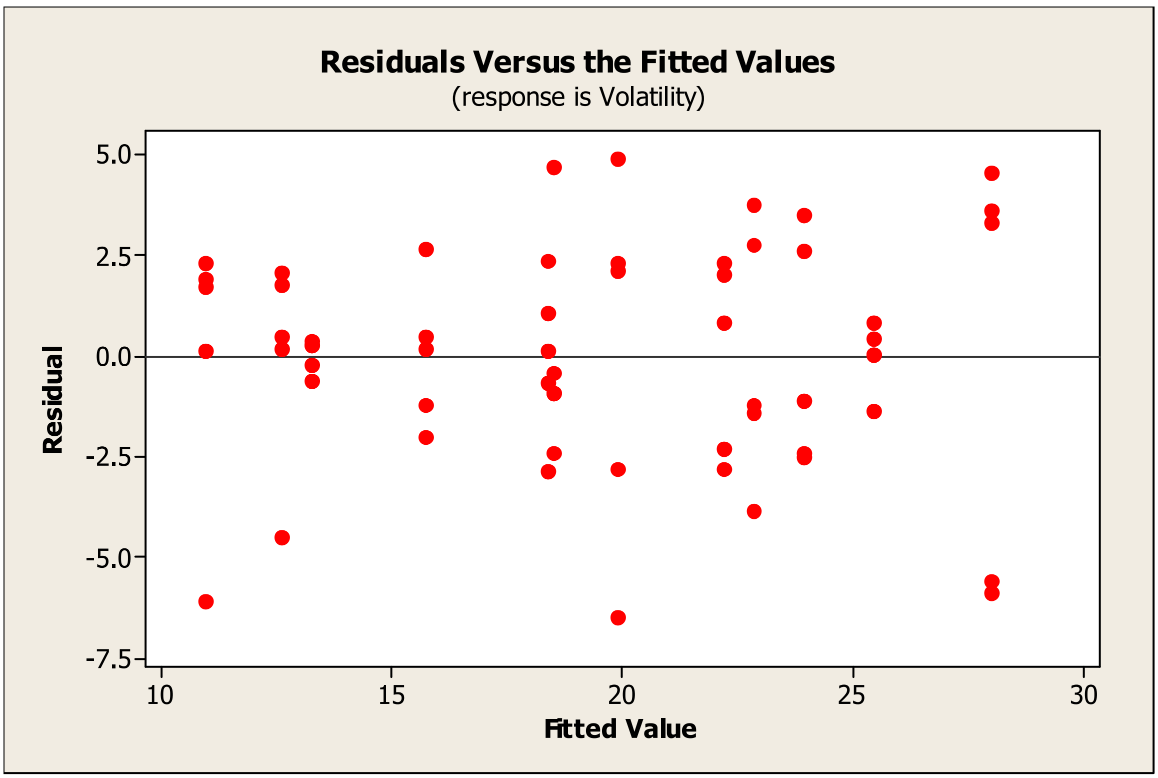

Boxplot of High-school, Undergrad+
Data
100
90
80
70
60
50
40
30
High-school
Undergrad+

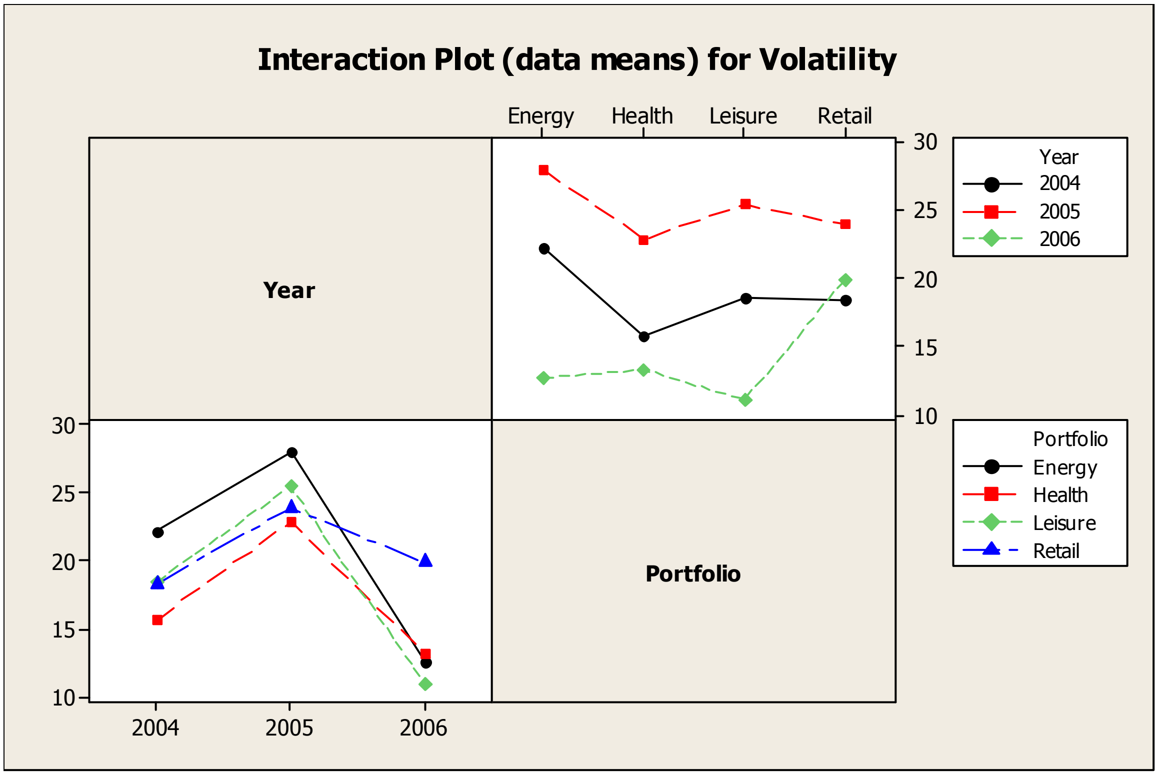
Appendix B

Two-way ANOVA: Volatility versus Year, Portfolio

Source	DF	SS	MS	F	P
Year		1191.58	595.792	66.82	0.000
Portfolio		146.55	48.851	5.48	0.003
Interaction					
Error		427.98	8.916		
Total		2031.31			

S = 2.986 R-Sq = 78.93% R-Sq(adj) = 74.10%





Tabulated statistics: Year, Portfolio

Rows: Year   Columns: Portfolio

	Energy	Health	Leisure	Retail	All
2004	22.20	15.74	18.52	18.36	18.71
	5	5	5	5	20
2005	27.98	22.84	25.46	23.92	25.05
	5	5	5	5	20
2006	12.62	13.24	10.98	19.90	14.19
	5	5	5	5	20
All	20.93	17.27	18.32	20.73	19.31
	15	15	15	15	60

Cell Contents: Volatility : Mean
Count

Appendix C

Model 1

Regression Analysis: Fat% versus Abdomen

The regression equation is
Fat% = - 29.7 + 0.516 Abdomen

Predictor	Coef	SE Coef	T	P
Constant	-29.685	4.774	-6.22	0.000
Abdomen	0.51556	0.05123	10.06	0.000

S = R-Sq = R-Sq(adj) = 67.2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression		2741.7	2741.7		
Residual Error		1299.3	27.1		
Total	49	4041.0			

Unusual Observations

Obs	Abdomen	Fat%	Fit	SE Fit	Residual	St Resid
39	148	33.800	46.670	2.963	-12.870	-3.01RX
41	126	33.100	35.379	1.896	-2.279	-0.47 X

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

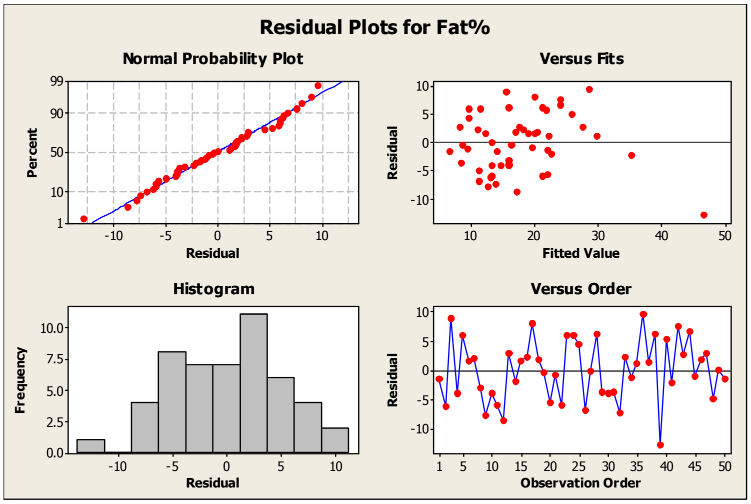
Predicted Values for New Observations

New Obs	Fit	SE Fit	95% CI	95% PI
1		0.743	(,)	(,)

Values of Predictors for New Observations

New Obs	Abdomen
1	90.0

Residual Plots for Fat%



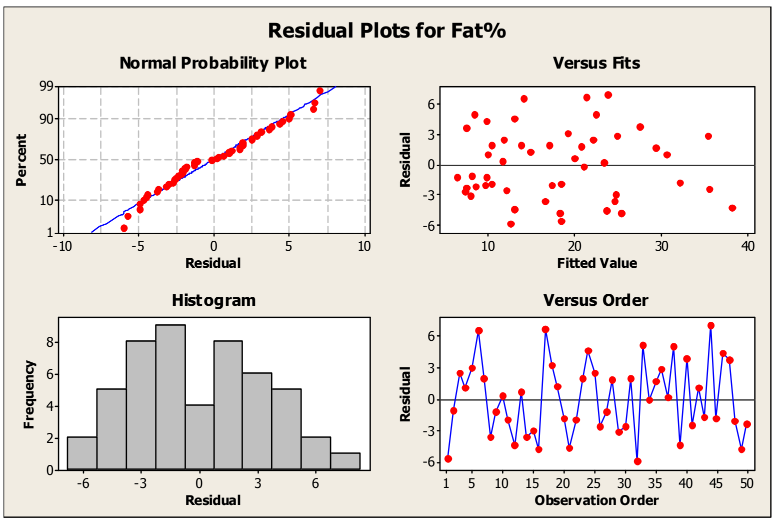
Model 2

Regression Analysis: Fat% versus Weight, Neck, Abdomen, Thigh, RegExer

```
The regression equation is
Fat% = - 39.9 - 0.210 Weight - 0.462 Neck + 0.868 Abdomen + 0.565 Thigh
       - 3.21 RegExer

Predictor       Coef  SE Coef      T      P     VIF
Constant      -39.89    12.78  -3.12  0.003
Weight      -0.20972  0.05488  -3.82  0.000  17.658
Neck         -0.4618   0.3523  -1.31  0.197   4.851
Abdomen      0.86845  0.09709   8.94  0.000   7.179
Thigh         0.5647   0.2334   2.42  0.020   8.885
RegExer       -3.214    1.228                 1.212

S = 3.68020   R-Sq = 85.3%   R-Sq(adj) = 83.6%

Analysis of Variance

Source          DF       SS      MS      F      P
Regression          3445.10
Residual Error       595.93   13.54
Total           49  4041.03

Source    DF   Seq SS
Weight     1  1512.14
Neck       1    66.47
Abdomen    1  1665.56
Thigh      1   108.18
RegExer    1    92.74


Unusual Observations

Obs  Weight   Fat%     Fit  SE Fit  Residual  St Resid
 39     363  33.800  38.217   2.471    -4.417     -1.62 X

X denotes an observation whose X value gives it large leverage.
```

```
Predicted Values for New Observations

New
Obs     Fit  SE Fit        95% CI                95% PI
  1           1.189  (      ,        )  (      ,        )

Values of Predictors for New Observations

New
Obs  Weight  Neck  Abdomen  Thigh  RegExer
  1     170  38.0     90.0   58.0     1.00
```

Residual Plots for Fat%



Model 3

Regression Analysis: Fat% versus Age, Weight, ...

```
The regression equation is
Fat% = - 32.0 + 0.0716 Age - 0.174 Weight - 0.022 Height - 0.537 Neck
       + 0.048 Chest + 0.842 Abdomen - 0.229 Hip + 0.700 Thigh - 2.90 RegExer
```

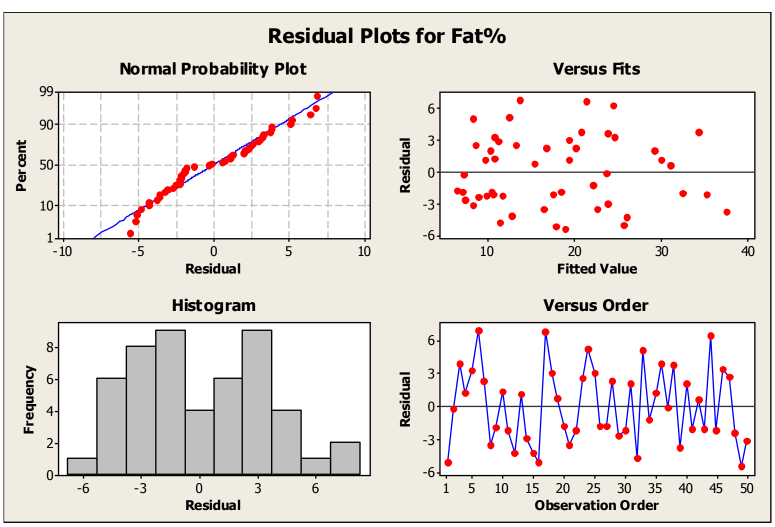
Predictor	Coef	SE Coef	T	P	VIF
Constant	-31.96	23.83	-1.34	0.188	
Age	0.07158	0.08447	0.85	0.402	1.729
Weight	-0.17441	0.07523	-2.32	0.026	31.490
Height	-0.0220	0.1123	-0.20	0.845	1.738
Neck	-0.5365	0.3946	-1.36	0.182	5.775
Chest	0.0484	0.1711	0.28	0.779	11.322
Abdomen	0.8424	0.1598	5.27	0.000	18.443
Hip	-0.2289	0.2650	-0.86	0.393	26.404
Thigh	0.6998	0.2798	2.50	0.017	12.124
RegExer	-2.904	1.303			1.293

```
S = 3.77809   R-Sq = 85.9%   R-Sq(adj) = 82.7%
```

```
Analysis of Variance
```

```
Source           DF       SS       MS       F      P
Regression           3470.08
Residual Error        570.96   14.27
Total            49  4041.03

Source    DF   Seq SS
Age        1  1081.71
Weight     1   978.64
Height     1   213.28
Neck       1     5.83
Chest      1   314.50
Abdomen    1   686.24
Hip        1     6.65
Thigh      1   112.30
RegExer    1    70.93


Unusual Observations

Obs   Age    Fat%     Fit  SE Fit  Residual  St Resid
 42  44.0  31.700  31.121   3.652     0.579      0.60 X

X denotes an observation whose X value gives it large leverage.

Predicted Values for New Observations

New
Obs     Fit  SE Fit       95% CI            95% PI
  1           3.006  (     ,       )  (       ,      )X

X denotes a point that is an outlier in the predictors.


Values of Predictors for New Observations

New
Obs   Age  Weight  Height  Neck  Chest  Abdomen   Hip  Thigh  RegExer
  1  25.0     170    70.0  38.0   95.0     90.0  90.0   58.0     1.00
```

Residual Plots for Fat%



Standard Normal Distribution

P(Z < z) **(z negative)**

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	**z**
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) **(z positive)**

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

	t_α										
	$\alpha = P(t > t_\alpha)$ = one-tail probability										
df	**0.100**	**0.050**	**0.025**	**0.010**	**0.009**	**0.008**	**0.005**	**0.001**	**0.0005**	**0.0004**	**0.0001**
1	3.08	6.31	12.71	31.82	35.36	39.78	63.66	318.31	636.62	837.66	3183.10
2	1.89	2.92	4.30	6.96	7.35	7.81	9.92	22.33	31.60	36.25	70.70
3	1.64	2.35	3.18	4.54	4.72	4.93	5.84	10.21	12.92	14.18	22.20
4	1.53	2.13	2.78	3.75	3.87	4.01	4.60	7.17	8.61	9.25	13.03
5	1.48	2.02	2.57	3.36	3.46	3.57	4.03	5.89	6.87	7.29	9.68
6	1.44	1.94	2.45	3.14	3.23	3.32	3.71	5.21	5.96	6.28	8.02
7	1.41	1.89	2.36	3.00	3.07	3.16	3.50	4.79	5.41	5.67	7.06
8	1.40	1.86	2.31	2.90	2.97	3.04	3.36	4.50	5.04	5.26	6.44
9	1.38	1.83	2.26	2.82	2.89	2.96	3.25	4.30	4.78	4.98	6.01
10	1.37	1.81	2.23	2.76	2.83	2.89	3.17	4.14	4.59	4.77	5.69
11	1.36	1.80	2.20	2.72	2.78	2.84	3.11	4.02	4.44	4.60	5.45
12	1.36	1.78	2.18	2.68	2.74	2.80	3.05	3.93	4.32	4.47	5.26
13	1.35	1.77	2.16	2.65	2.71	2.77	3.01	3.85	4.22	4.37	5.11
14	1.35	1.76	2.14	2.62	2.68	2.74	2.98	3.79	4.14	4.28	4.99
15	1.34	1.75	2.13	2.60	2.66	2.71	2.95	3.73	4.07	4.21	4.88
16	1.34	1.75	2.12	2.58	2.64	2.69	2.92	3.69	4.01	4.15	4.79
17	1.33	1.74	2.11	2.57	2.62	2.67	2.90	3.65	3.97	4.09	4.71
18	1.33	1.73	2.10	2.55	2.60	2.66	2.88	3.61	3.92	4.04	4.65
19	1.33	1.73	2.09	2.54	2.59	2.64	2.86	3.58	3.88	4.00	4.59
20	1.33	1.72	2.09	2.53	2.58	2.63	2.85	3.55	3.85	3.97	4.54
21	1.32	1.72	2.08	2.52	2.57	2.62	2.83	3.53	3.82	3.93	4.49
22	1.32	1.72	2.07	2.51	2.56	2.61	2.82	3.50	3.79	3.91	4.45
23	1.32	1.71	2.07	2.50	2.55	2.60	2.81	3.48	3.77	3.88	4.42
24	1.32	1.71	2.06	2.49	2.54	2.59	2.80	3.47	3.75	3.85	4.38
25	1.32	1.71	2.06	2.49	2.53	2.58	2.79	3.45	3.73	3.83	4.35
26	1.31	1.71	2.06	2.48	2.53	2.58	2.78	3.43	3.71	3.81	4.32
27	1.31	1.70	2.05	2.47	2.52	2.57	2.77	3.42	3.69	3.79	4.30
28	1.31	1.70	2.05	2.47	2.51	2.56	2.76	3.41	3.67	3.78	4.28
29	1.31	1.70	2.05	2.46	2.51	2.56	2.76	3.40	3.66	3.76	4.25
30	1.31	1.70	2.04	2.46	2.50	2.55	2.75	3.39	3.65	3.75	4.23
31	1.31	1.70	2.04	2.45	2.50	2.55	2.74	3.37	3.63	3.73	4.22
32	1.31	1.69	2.04	2.45	2.49	2.54	2.74	3.37	3.62	3.72	4.20
33	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.36	3.61	3.71	4.18
34	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.35	3.60	3.70	4.17
35	1.31	1.69	2.03	2.44	2.48	2.53	2.72	3.34	3.59	3.69	4.15
36	1.31	1.69	2.03	2.43	2.48	2.53	2.72	3.33	3.58	3.68	4.14
37	1.30	1.69	2.03	2.43	2.48	2.52	2.72	3.33	3.57	3.67	4.13
38	1.30	1.69	2.02	2.43	2.47	2.52	2.71	3.32	3.57	3.66	4.12
39	1.30	1.68	2.02	2.43	2.47	2.52	2.71	3.31	3.56	3.65	4.10
40	1.30	1.68	2.02	2.42	2.47	2.52	2.70	3.31	3.55	3.65	4.09
50	1.30	1.68	2.01	2.40	2.45	2.49	2.68	3.26	3.50	3.59	4.01
60	1.30	1.67	2.00	2.39	2.43	2.48	2.66	3.23	3.46	3.55	3.96
100	1.29	1.66	1.98	2.36	2.41	2.45	2.63	3.17	3.39	3.47	3.86
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.58	3.10	3.30	3.38	3.73

Chi-square distribution

$\chi^2_{\alpha;\nu}$

$\alpha = P(\chi^2 \geq \chi^2_{\alpha;\nu})$

ν	0.995	0.100	0.050	0.025	0.010	0.005	0.004	0.003	0.002	0.001	0.0005	0.0001
1	0.00	2.71	3.84	5.02	6.63	7.88	8.28	8.81	9.55	10.83	12.12	15.14
2	0.01	4.61	5.99	7.38	9.21	10.60	11.04	11.62	12.43	13.82	15.20	18.42
3	0.07	6.25	7.81	9.35	11.34	12.84	13.32	13.93	14.80	16.27	17.73	21.11
4	0.21	7.78	9.49	11.14	13.28	14.86	15.37	16.01	16.92	18.47	20.00	23.51
5	0.41	9.24	11.07	12.83	15.09	16.75	17.28	17.96	18.91	20.52	22.11	25.74
6	0.68	10.64	12.59	14.45	16.81	18.55	19.10	19.80	20.79	22.46	24.10	27.86
7	0.99	12.02	14.07	16.01	18.48	20.28	20.85	21.58	22.60	24.32	26.02	29.88
8	1.34	13.36	15.51	17.53	20.09	21.95	22.55	23.30	24.35	26.12	27.87	31.83
9	1.73	14.68	16.92	19.02	21.67	23.59	24.20	24.97	26.06	27.88	29.67	33.72
10	2.16	15.99	18.31	20.48	23.21	25.19	25.81	26.61	27.72	29.59	31.42	35.56
11	2.60	17.28	19.68	21.92	24.72	26.76	27.40	28.22	29.35	31.26	33.14	37.37
12	3.07	18.55	21.03	23.34	26.22	28.30	28.96	29.79	30.96	32.91	34.82	39.13
13	3.57	19.81	22.36	24.74	27.69	29.82	30.49	31.35	32.54	34.53	36.48	40.87
14	4.07	21.06	23.68	26.12	29.14	31.32	32.00	32.88	34.09	36.12	38.11	42.58
15	4.60	22.31	25.00	27.49	30.58	32.80	33.50	34.39	35.63	37.70	39.72	44.26
16	5.14	23.54	26.30	28.85	32.00	34.27	34.98	35.89	37.15	39.25	41.31	45.92
17	5.70	24.77	27.59	30.19	33.41	35.72	36.44	37.37	38.65	40.79	42.88	47.57
18	6.26	25.99	28.87	31.53	34.81	37.16	37.89	38.83	40.14	42.31	44.43	49.19
19	6.84	27.20	30.14	32.85	36.19	38.58	39.33	40.29	41.61	43.82	45.97	50.80
20	7.43	28.41	31.41	34.17	37.57	40.00	40.76	41.73	43.07	45.31	47.50	52.39
21	8.03	29.62	32.67	35.48	38.93	41.40	42.17	43.16	44.52	46.80	49.01	53.96
22	8.64	30.81	33.92	36.78	40.29	42.80	43.58	44.58	45.96	48.27	50.51	55.52
23	9.26	32.01	35.17	38.08	41.64	44.18	44.98	45.99	47.39	49.73	52.00	57.07
24	9.89	33.20	36.42	39.36	42.98	45.56	46.37	47.39	48.81	51.18	53.48	58.61
25	10.52	34.38	37.65	40.65	44.31	46.93	47.75	48.78	50.22	52.62	54.95	60.14
26	11.16	35.56	38.89	41.92	45.64	48.29	49.12	50.17	51.63	54.05	56.41	61.66
27	11.81	36.74	40.11	43.19	46.96	49.64	50.48	51.55	53.02	55.48	57.86	63.16
28	12.46	37.92	41.34	44.46	48.28	50.99	51.84	52.92	54.41	56.89	59.30	64.66
29	13.12	39.09	42.56	45.72	49.59	52.34	53.19	54.28	55.79	58.30	60.73	66.15
30	13.79	40.26	43.77	46.98	50.89	53.67	54.54	55.64	57.17	59.70	62.16	67.63
35	17.19	46.06	49.80	53.20	57.34	60.27	61.19	62.35	63.95	66.62	69.20	74.93
40	20.71	51.81	55.76	59.34	63.69	66.77	67.72	68.94	70.62	73.40	76.09	82.06
50	27.99	63.17	67.50	71.42	76.15	79.49	80.53	81.84	83.66	86.66	89.56	95.97
100	67.33	118.50	124.34	129.56	135.81	140.17	141.52	143.23	145.58	149.45	153.17	161.32
150	109.14	172.58	179.58	185.80	193.21	198.36	199.95	201.96	204.72	209.26	213.61	223.11
200	152.24	226.02	233.99	241.06	249.45	255.26	257.06	259.33	262.43	267.54	272.42	283.06
224	173.24	251.52	259.91	267.35	276.16	282.27	284.15	286.53	289.79	295.14	300.26	311.39
250	196.16	279.05	287.88	295.69	304.94	311.35	313.32	315.81	319.23	324.83	330.18	341.83
500	422.30	540.93	553.13	563.85	576.49	585.21	587.89	591.26	595.88	603.45	610.65	626.24

Fisher's F distribution

$F_{.05}$

$P(F \geq F_{.05}) = 0.050$

ν_1

ν_2	1	2	3	4	5	6	7	8	9	10	11	12
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.0	243.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	2.11	2.08
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	2.10	2.07
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	2.09	2.06
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	2.08	2.05
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.07	2.04
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80