École de gestion TELFER School of Management

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++ADM 2304 – Final Examination
APPLIED STATISTICAL METHODS IN BUSINESS
29 April 2010

NAME (please print): ____

STUDENT NUMBER: ______________________________

SECTION (please circle one): M N P Q R

Time allowed: **3 hours**
Question booklet: **10 pages (including front cover)**
Appendices: **5 pages of appendices and 5 pages of statistical tables**

Instructions:

1. ***Complete all tests with hypotheses, test statistic, critical value, decision, and conclusion. Use the .05 level of significance unless otherwise indicated.***
2. Calculators and one sheet of notes (8.5 x 14 in.) are allowed.
3. You must hand back all exam materials (exam question booklet, appendices and tables), but please keep your personal sheet of notes for future use.

Question	Value	Mark
1	8	
2	5	
3	6	
4	13	
5	14	
6	14	
Total	**60**	

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ______________________________

Question 1. [8 marks]

The Community Care Access Center (CCAC) is responsible for determining which client waiting for placement in long term care receives the next available bed. Recently they have received a fair bit of criticism from the hospitals with the two smaller hospitals – Queensway-Carleton (QCH) and Montfort (MON) - suggesting that the Ottawa Hospital (TOH) is getting more access than they are and TOH suggesting that it is getting less access. The CCAC decided to determine if either group's claim is true by looking at the wait times from the date a client becomes eligible for long term care to the date they get placed for TOH versus the other two hospitals combined. Do a single test to see if either claim is true. "QCHMON Wait Time" represents the wait time for MON and QCH combined while "TOH Wait Time" represents the wait time for TOH. Make sure you justify the use of the test you choose. Appendix A gives some Minitab analyses of the data.

a) What is your choice of test? Explain briefly.
[1]

b) Perform the parametric test. Use the .05 level of significance.
[4]

c) Now perform the non-parametric test.
[3]

Question 2. [5 marks]

The CCAC also decided to look at the client choice of type of bed or room in the long-term facility. Clients are categorized based on if they choose a private bed, a ward bed or are willing to take either. The CCAC hypothesized that there should be no difference between the three hospitals (MON, QCH and TOH) in terms of these choices. The breakdown of data is as follows:

	Private	Ward	Either	Total
MON	**216**	**175**	**83**	474
QCH	**275**	**241**	**147**	663
TOH	**444**	**778**	**657**	1879
Total	935	1194	887	3016

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

     Private    Ward  Either  Total
    1     216     175      83    474
      ______  187.65  139.40
      ______   0.853  22.821

    2     275     241     147    663
      205.54  262.47  194.99
      23.474   1.757  11.810

    3     444     778     657   1879
      ______  743.87  ______
      ______   1.566  ______

Total     935    1194     887   3016
```

Determine whether client choice differs by hospital. What is the approximate p-value?

Question 3 [6 marks]

Finally, the CCAC decided to look at preferences among the 28 long-term facilities in the Ottawa region.

The five largest facilities have the following relative capacities, with the 23 smaller facilities sharing the remaining 61% capacity.

PR	11%
GR	8%
SA	8%
PC	6%
GA	6%

The facility preference data were as follows:

Facility	Preference
PR	422
GR	304
SA	293
PC	239
GA	233
Other	2036

a) Test whether or not the relative preference for each facility is consistent with the relative capacity.

[5]

b) Is there anything about the relative capacity data to suggest that your conclusion above might be suspect?

[1]

Question 4. [13 marks]

The leading digits of numbers occurring in real life were carefully observed and an empirical law was discovered by a physicist named Benford. For example, the numbers 123, 1056, 12967, 130546 all have '1' as the leading digit.

According to this law, the leading digit '1' occurs 30.6% (and not 11.11% as expected) of the times in numbers occurring in real life. The other digits also have different percentage values according to this law. In this question only the leading digit '1' is considered.

Forensic auditors use Benford's law to flag tax returns which **could** be fraudulent. In a comparison of two samples of tax returns, the following data were obtained. Sample#1 was a simple random sample of typical high income tax returns and Sample#2 was a simple random sample of typical middle income tax returns. Suspected frauds are the returns which need auditing and in this example are based on the relative frequency of leading digit '1' being other than $0.306 \pm .030$.

	Sample #1 (high income)	**Sample #2 (middle income)**
X- Suspected number of frauds:	**60**	**30**
n - Total number Tax Returns:	**450**	**500**

a) From the above data, test the hypothesis that the population proportion of suspected fraudulent high income tax returns is greater than 11.5%. Use a level of significance of 5%.

[3]

b) For the above test, we assume that ____________________________ is normally distributed. To determine whether this is valid, we check if

____ .

[2] (Fill in the blanks above.)

c) Calculate a one-sided 95% confidence interval or the ‘bound’ for the population proportion which is appropriate for the test in (a).

[2]

d) How big a sample of middle income returns would we need if we wanted to calculate a 95% confidence interval with a margin of error of plus-or-minus 1%?

[2]

e) Now test the hypothesis that the two population proportions are different. Use a 5% level of significance.

[4]

Question 5. [14 marks]

A large electronic service company collected data on the 'Training Time' (in days) given to its technicians and the 'Time' (in minutes) required to repair different types of defective equipment. The equipment types are 'E1:Fax Machines', 'E2:Photo-Copiers' 'E3: Computers', and 'E4: Televisions'.

The relevant data and Minitab output are given in Appendix B. Perform all tests using the .05 level of significance.

a) Complete the ANOVA table up to the 'MS' Column.
[3]

Source	DF	SS	MS
TrainingTime		593.06	
Equipment		2396.53	
Interaction		418.06	
Error	24	1683.33	70.139
Total		5090.97	

b) What is the estimated standard deviation "s" of the error terms?
[1]

c) Calculate the adjusted R^2.
[1]

d) Comment on the presence or absence of interaction, based on the plot. Explain briefly.
[1]

e) Test whether the effect of training time on service time depends on the equipment type.

[3]

f) Calculate the margin of error for comparing average service times on different equipment types, using the Bonferroni method of multiple comparisons. Use the closest critical value on the appropriate table.

[3]

g) Explain if there is a real difference in the average service time comparing technicians working on TVs with those working on computers, or between fax machines and photocopiers.

[2]

Question 6. [14 marks]

The objective of the Study on the Efficacy of Nosocomial Infection Control (SENIC Project, **American Journal of Epidemiology 111** (1980), p. 465-653) was to determine whether infection surveillance and control programs have reduced the rates of nosocomial (hospital-acquired) infection in hospitals. The analyses are found in Appendix C.

The variables are:

ID	Identification number of hospital
LoS	Average Length of Stay (days)
Age	Average age of patients (years)
InfectRisk	Average estimated probability of acquiring infection in hospital (percentage)
CultRat	Ratio of number of cultures to number of patients without hospital-acquired infection (percentage)
XrayRat	Ratio of number of X-rays to number of patients without pneumonia (percentage)
NumBeds	Average number of beds
MedSch	1 – if hospital is associated with a medical school, 0 -- otherwise
West	1 – if hospital is in the West Region, 0 -- otherwise
AvgCens	Average number of patients per day during study period
NumbNurses	Average number of nurses
Available	Percentage of 35 potential services and services provided by hospital

a) Looking at the residual plot from Model 1, what potential problem do you see with the regression model assumptions? What assumption might be violated?
[1]

b) Has this problem been solved in Model 2? Explain briefly.
[1]

c) Test whether Model 2 is helpful for predicting the Infection Risk.
[3]

d) Test whether the ratio of the number of cultures to the number of patients without hospital-acquired infection has an effect on the Infection Risk.

[3]

e) How would you interpret the value of the coefficient of the West variable?

[2]

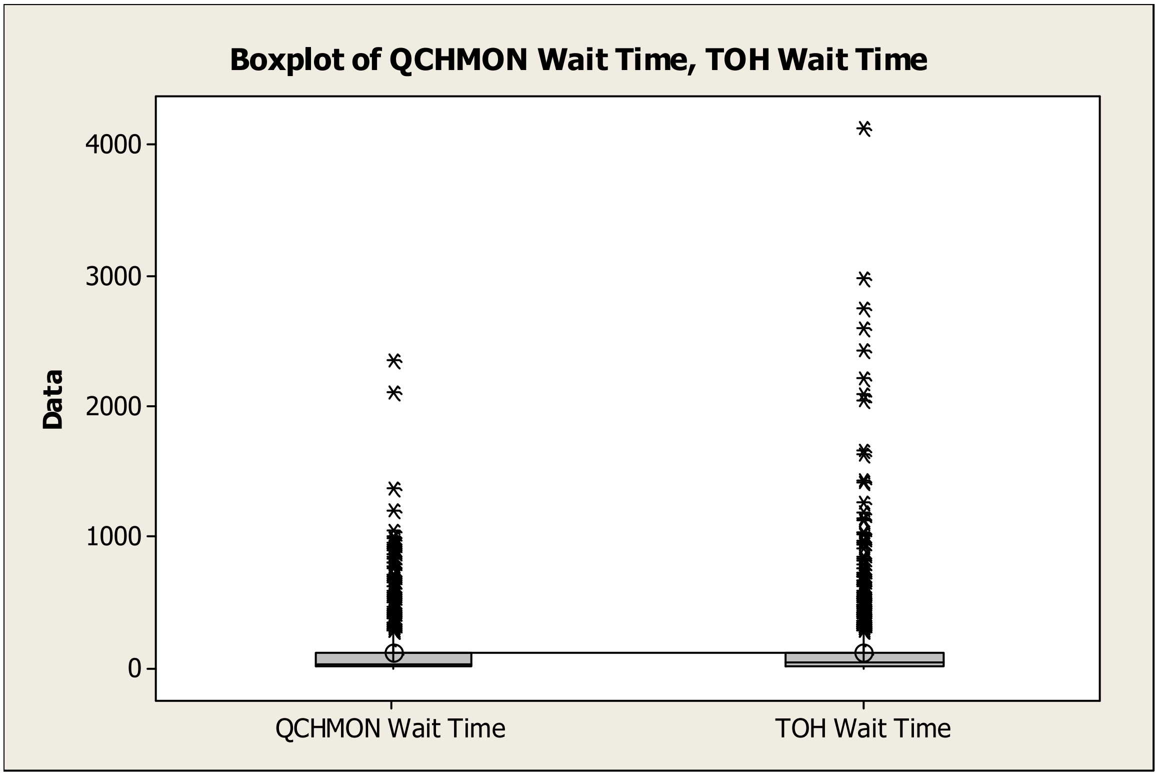
f) Why might the interpretation of the number of beds coefficient be problematic?

[1]

g) Using Model 2, calculate the **99%** interval for predicting the infection rate for the outlying hospital that was dropped from Model 1 to Model 2. Does this interval cover the actual infection rate for this hospital?

[3]

Appendix A



Two-Sample T-Test and CI: QCHMON Wait Time, TOH Wait Time

```
Two-sample T for QCHMON Wait Time vs TOH Wait Time

                     N    Mean   StDev  SE Mean
QCHMON Wait Time    783    115     217    7.8
TOH Wait Time      1374    124    270     7.3
```

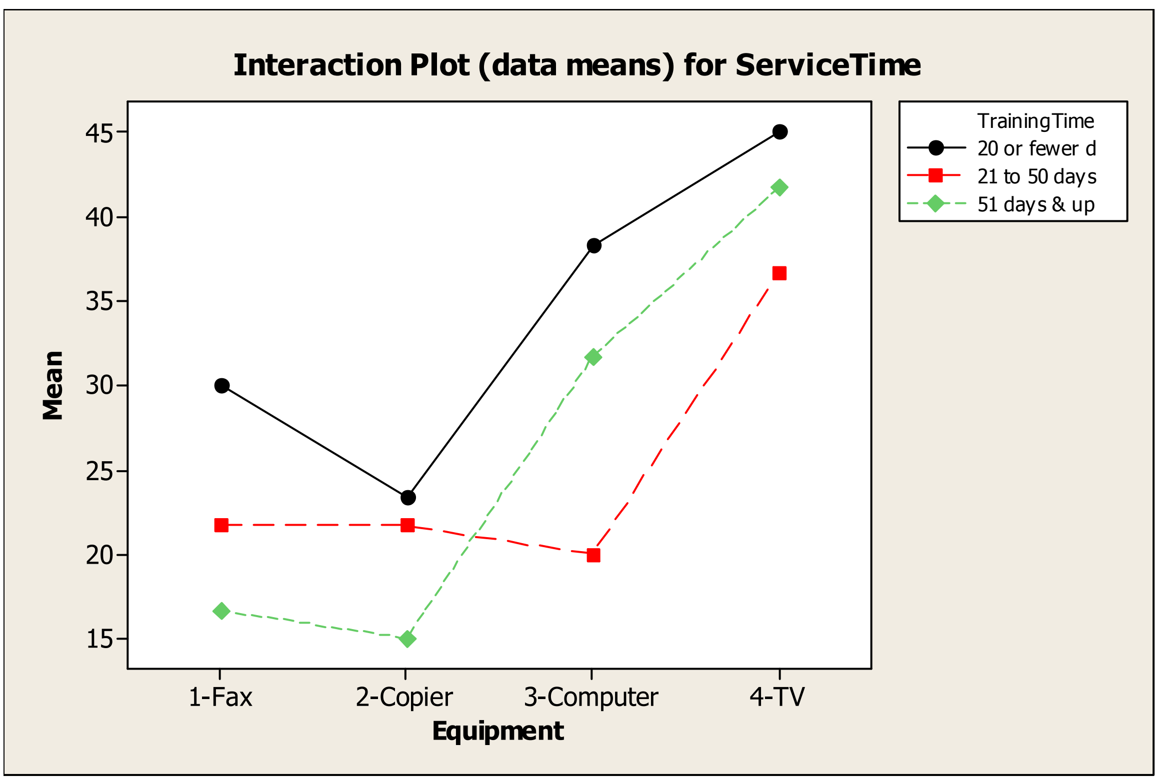
Mann-Whitney Test and CI: QCHMON Wait Time, TOH Wait Time

```
                     N  Median
QCHMON Wait Time   783   33.00
TOH Wait Time     1374   41.00

Point estimate for ETA1-ETA2 is -2.00
95.0 Percent CI for ETA1-ETA2 is (-4.00,1.01)
W = 827270.0
Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.2061
The test is significant at 0.2061 (adjusted for ties)
```

Appendix B

Tabulated statistics: TrainingTime, Equipment

```
Rows: TrainingTime   Columns: Equipment

                  1-Fax  2-Copier  3-Computer   4-TV    All

20 or fewer d     30.00     23.33       38.33  45.00  34.17

                     25        20          30     50
                     30        20          45     45
                     35        30          40     40

21 to 50 days     21.67     21.67       20.00  36.67  25.00

                     20        10          20     15
                     25        30          10     50
                     20        25          30     45

51 days & up      16.67     15.00       31.67  41.67  26.25


                     25        10          25     40
                     10        15          35     40
                     15        20          35     45

All               22.78     20.00       30.00  41.11  28.47

Cell Contents:   ServiceTime  :  Mean
                 ServiceTime  :  DATA
```



Appendix C

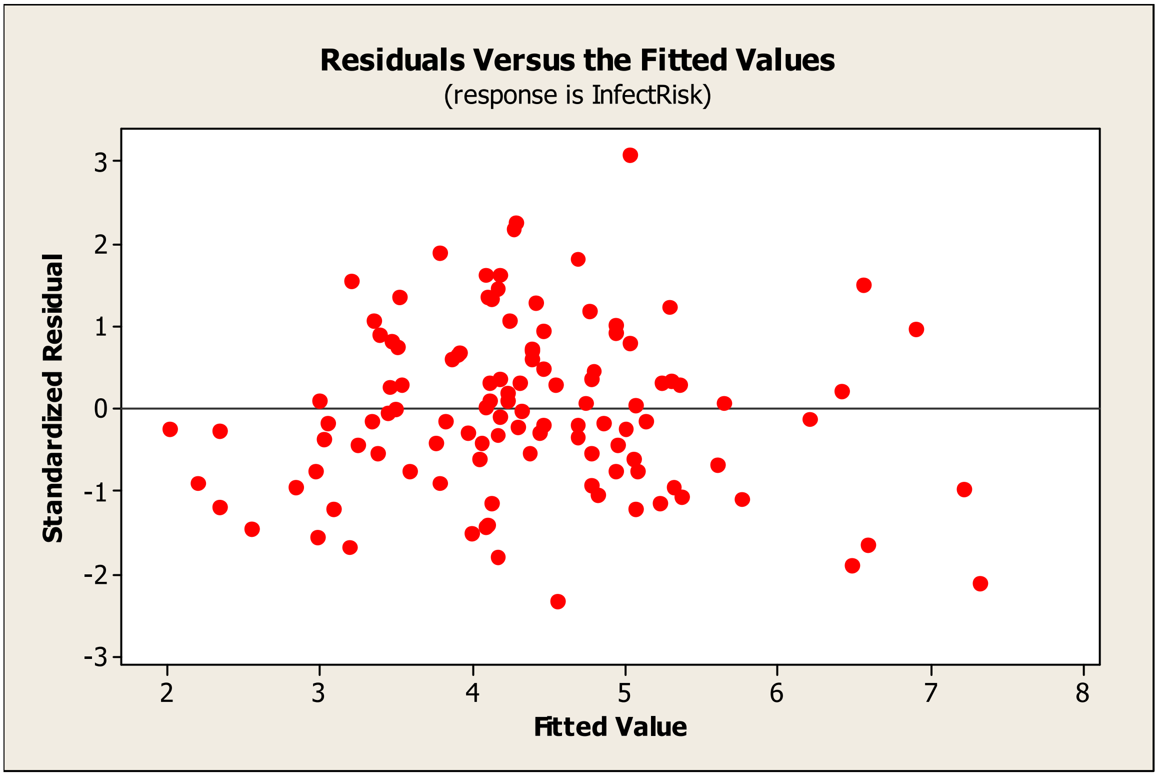
The following table shows an extract of the data:

ID	Infect Risk	Culture Rat	Xray Rat	Num Beds	Med Sch	Avg Cens	Numb Nurses	Avail-able	LoS	Age	West
51	5.6	20.3	82	252	0	207	251	51.4	11.48	57.6	0
52	4.3	11.6	42.6	620	0	413	420	71.4	9.23	51.6	0
53	7.6	16.6	97.9	535	0	330	273	51.4	11.41	61.1	0
54	7.8	52.4	105.3	157	0	115	76	31.4	12.07	43.7	0
55	3.1	8.4	56.2	76	0	39	44	31.4	8.63	54	0

Model 1.

Regression Analysis: InfectRisk versus CultRat, XrayRat, ...

```
The regression equation is
InfectRisk = - 0.93 + 0.0514 CultRat + 0.0113 XrayRat - 0.00249 NumBeds
             - 0.618 MedSch + 0.00317 AvgCens + 0.00148 NumbNurses
             + 0.0180 Available + 0.215 LoS + 0.0084 Age + 0.811 West

Predictor         Coef   SE Coef       T       P   VIF
Constant        -0.931     1.159   -0.80   0.424
CultRat        0.05136   0.01042   _____  ______   1.5
XrayRat       0.011322  0.005251    2.16   0.033   1.4
NumBeds      -0.002492  0.002619   -0.95   0.344  34.0
MedSch         -0.6181    0.3199   -1.93   0.056   1.8
AvgCens       0.003169  0.003424    0.93   0.357  36.9
NumbNurses    0.001477  0.001690    0.87   0.384   7.4
Available     0.017974  0.009784    1.84   0.069   2.9
LoS            0.21457   0.06748    3.18   0.002   2.2
Age            0.00843   0.02143    0.39   0.695   1.2
West            0.8107    0.2678    3.03   0.003   1.2

S = 0.917301   R-Sq = 57.4%   R-Sq(adj) = 53.2%

Analysis of Variance

Source           DF       SS      MS      F       P
Regression       10  115.553  11.555  13.73  ______
Residual Error  102   85.827   0.841
Total           112  201.380

Unusual Observations

Obs  CultRat  InfectRisk     Fit  SE Fit  Residual  St Resid
  8     60.5      5.4000  6.5935  0.5612   -1.1935    -1.64 X
 35     11.4      6.3000  4.2827  0.1733    2.0173     2.24R
 47     17.2      6.5000  7.2222  0.5407   -0.7222    -0.97 X
 53     16.6      7.6000  5.0330  0.3757    2.5670     3.07R
 96     27.0      2.5000  4.5638  0.2312   -2.0638    -2.32R
100     16.4      6.2000  4.2744  0.2592    1.9256     2.19R
112     26.4      5.9000  7.3268  0.6214   -1.4268    -2.11RX

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large influence.
```



Model 2.

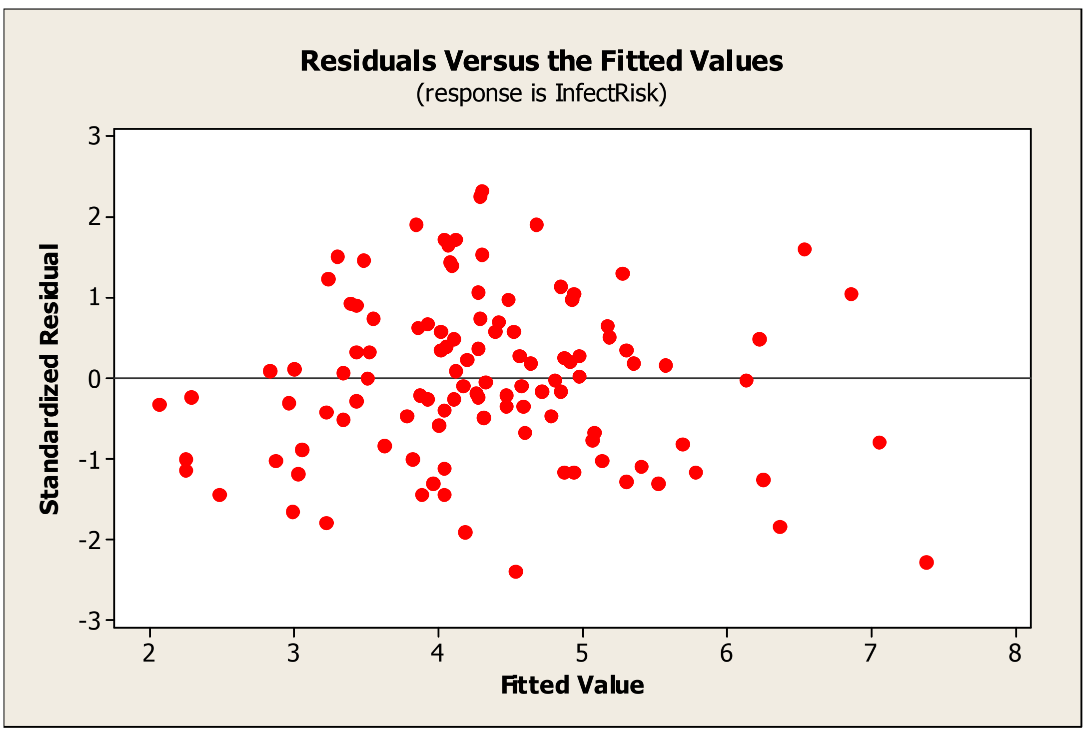
Regression Analysis: InfectRisk versus CultRat, XrayRat, ...

```
The regression equation is
InfectRisk = - 0.22 + 0.0513 CultRat + 0.00993 XrayRat - 0.00534 NumBeds
             - 0.563 MedSch + 0.00599 AvgCens + 0.00183 NumbNurses
             + 0.0210 Available + 0.203 LoS - 0.0016 Age + 0.840 West


112 cases used, 1 cases contain missing values
```

Predictor	Coef	SE Coef	T	P	VIF
Constant	-0.220	1.132	-0.19	0.846	
CultRat	0.051314	0.009979	_____	______	1.5
XrayRat	0.009931	0.005047	1.97	0.052	1.4
NumBeds	-0.005342	0.002661	-2.01	0.047	37.5
MedSch	-0.5627	0.3067	-1.83	0.070	1.8
AvgCens	0.005991	0.003395	1.76	0.081	39.3
NumbNurses	0.001828	0.001622	1.13	0.263	7.4
Available	0.020985	0.009415	2.23	0.028	3.0
LoS	0.20277	0.06471	3.13	0.002	2.2
Age	-0.00162	0.02076	-0.08	0.938	1.2
West	0.8405	0.2566	3.28	0.001	1.2

```
S = 0.878282   R-Sq = 59.2%   R-Sq(adj) = 55.1%
```

```
Analysis of Variance

Source           DF       SS      MS      F       P
Regression       10  112.846  11.285  14.63  _____
Residual Error  101   77.909   0.771
Total           111  190.755

Unusual Observations

Obs  CultRat  InfectRisk     Fit  SE Fit  Residual  St Resid
  8     60.5      5.4000  6.2556  0.5475   -0.8556     -1.25 X
 35     11.4      6.3000  4.2996  0.1660    2.0004      2.32R
 47     17.2      6.5000  7.0551  0.5203   -0.5551     -0.78 X
 96     27.0      2.5000  4.5399  0.2215   -2.0399     -2.40R
100     16.4      6.2000  4.2927  0.2483    1.9073      2.26R
112     26.4      5.9000  7.3727  0.5951   -1.4727     -2.28RX

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large influence.

Predicted Values for New Observations

New
Obs     Fit  SE Fit       95% CI              95% PI
  1  4.5155  0.3943  (3.7333, 5.2978)  (2.6057, 6.4254)

Values of Predictors for New Observations

New
Obs  CultRat  XrayRat  NumBeds    MedSch  AvgCens  NumbNurses  Available   LoS
  1     16.6     97.9      535  0.000000      330         273       51.4  11.4
New
Obs   Age      West
  1  61.1  0.000000
```



Standard Normal Distribution

P(Z < z) (z negative)

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	**z**
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) (z positive)

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

Φ	t_Φ $\Phi = P(t > t_\Phi)$ 0.050	0.025	0.010	0.008	0.005	0.004	0.003	0.002	0.001	0.0004
1	6.31	12.71	31.82	39.78	63.66	79.57	106.10	159.15	318.31	795.77
2	2.92	4.30	6.96	7.81	9.92	11.11	12.85	15.76	22.33	35.33
3	2.35	3.18	4.54	4.93	5.84	6.32	6.99	8.05	10.21	13.94
4	2.13	2.78	3.75	4.01	4.60	4.91	5.32	5.95	7.17	9.13
5	2.02	2.57	3.36	3.57	4.03	4.26	4.57	5.03	5.89	7.21
6	1.94	2.45	3.14	3.32	3.71	3.90	4.15	4.52	5.21	6.22
7	1.89	2.36	3.00	3.16	3.50	3.67	3.89	4.21	4.79	5.62
8	1.86	2.31	2.90	3.04	3.36	3.51	3.70	3.99	4.50	5.22
9	1.83	2.26	2.82	2.96	3.25	3.39	3.57	3.83	4.30	4.94
10	1.81	2.23	2.76	2.89	3.17	3.30	3.47	3.72	4.14	4.73
11	1.80	2.20	2.72	2.84	3.11	3.23	3.39	3.62	4.02	4.57
12	1.78	2.18	2.68	2.80	3.05	3.17	3.33	3.55	3.93	4.44
13	1.77	2.16	2.65	2.77	3.01	3.13	3.28	3.49	3.85	4.34
14	1.76	2.14	2.62	2.74	2.98	3.09	3.23	3.44	3.79	4.26
15	1.75	2.13	2.60	2.71	2.95	3.06	3.20	3.39	3.73	4.18
16	1.75	2.12	2.58	2.69	2.92	3.03	3.17	3.36	3.69	4.12
17	1.74	2.11	2.57	2.67	2.90	3.00	3.14	3.33	3.65	4.07
18	1.73	2.10	2.55	2.66	2.88	2.98	3.11	3.30	3.61	4.02
19	1.73	2.09	2.54	2.64	2.86	2.96	3.09	3.27	3.58	3.98
20	1.72	2.09	2.53	2.63	2.85	2.95	3.07	3.25	3.55	3.95
21	1.72	2.08	2.52	2.62	2.83	2.93	3.06	3.23	3.53	3.91
22	1.72	2.07	2.51	2.61	2.82	2.92	3.04	3.21	3.50	3.88
23	1.71	2.07	2.50	2.60	2.81	2.90	3.03	3.20	3.48	3.86
24	1.71	2.06	2.49	2.59	2.80	2.89	3.01	3.18	3.47	3.83
25	1.71	2.06	2.49	2.58	2.79	2.88	3.00	3.17	3.45	3.81
26	1.71	2.06	2.48	2.58	2.78	2.87	2.99	3.16	3.43	3.79
27	1.70	2.05	2.47	2.57	2.77	2.86	2.98	3.15	3.42	3.78
28	1.70	2.05	2.47	2.56	2.76	2.86	2.97	3.14	3.41	3.76
29	1.70	2.05	2.46	2.56	2.76	2.85	2.96	3.13	3.40	3.74
30	1.70	2.04	2.46	2.55	2.75	2.84	2.96	3.12	3.39	3.73
31	1.70	2.04	2.45	2.55	2.74	2.83	2.95	3.11	3.37	3.72
32	1.69	2.04	2.45	2.54	2.74	2.83	2.94	3.10	3.37	3.70
33	1.69	2.03	2.44	2.54	2.73	2.82	2.94	3.09	3.36	3.69
34	1.69	2.03	2.44	2.54	2.73	2.82	2.93	3.09	3.35	3.68
35	1.69	2.03	2.44	2.53	2.72	2.81	2.93	3.08	3.34	3.67
36	1.69	2.03	2.43	2.53	2.72	2.81	2.92	3.08	3.33	3.66
37	1.69	2.03	2.43	2.52	2.72	2.80	2.92	3.07	3.33	3.65
38	1.69	2.02	2.43	2.52	2.71	2.80	2.91	3.06	3.32	3.64
39	1.68	2.02	2.43	2.52	2.71	2.80	2.91	3.06	3.31	3.64
40	1.68	2.02	2.42	2.52	2.70	2.79	2.90	3.05	3.31	3.63
50	1.68	2.01	2.40	2.49	2.68	2.76	2.87	3.02	3.26	3.57
100	1.66	1.98	2.36	2.45	2.63	2.71	2.81	2.95	3.17	3.46
200	1.65	1.97	2.35	2.43	2.60	2.68	2.78	2.91	3.13	3.40
1000	1.65	1.96	2.33	2.41	2.58	2.66	2.75	2.88	3.10	3.36

Chi-Square Distribution

χ^2_α

$\alpha = P(\chi^2 \geq \chi^2_\alpha)$

ν	0.100	0.050	0.025	0.010	0.005	0.001	0.0001	0.00001	0.000001	0.0000005
1	2.71	3.84	5.02	6.63	7.88	10.83	15.14	19.51	23.93	25.26
2	4.61	5.99	7.38	9.21	10.60	13.82	18.42	23.03	27.63	29.02
3	6.25	7.81	9.35	11.34	12.84	16.27	21.11	25.90	30.66	32.09
4	7.78	9.49	11.14	13.28	14.86	18.47	23.51	28.47	33.38	34.84
5	9.24	11.07	12.83	15.09	16.75	20.52	25.74	30.86	35.89	37.39
6	10.64	12.59	14.45	16.81	18.55	22.46	27.86	33.11	38.26	39.79
7	12.02	14.07	16.01	18.48	20.28	24.32	29.88	35.26	40.52	42.09
8	13.36	15.51	17.53	20.09	21.95	26.12	31.83	37.33	42.70	44.30
9	14.68	16.92	19.02	21.67	23.59	27.88	33.72	39.34	44.81	46.43
10	15.99	18.31	20.48	23.21	25.19	29.59	35.56	41.30	46.86	48.51
11	17.28	19.68	21.92	24.72	26.76	31.26	37.37	43.21	48.87	50.54
12	18.55	21.03	23.34	26.22	28.30	32.91	39.13	45.08	50.83	52.53
13	19.81	22.36	24.74	27.69	29.82	34.53	40.87	46.91	52.75	54.47
14	21.06	23.68	26.12	29.14	31.32	36.12	42.58	48.72	54.64	56.38
15	22.31	25.00	27.49	30.58	32.80	37.70	44.26	50.49	56.49	58.26
16	23.54	26.30	28.85	32.00	34.27	39.25	45.92	52.24	58.32	60.12
17	24.77	27.59	30.19	33.41	35.72	40.79	47.57	53.97	60.13	61.95
18	25.99	28.87	31.53	34.81	37.16	42.31	49.19	55.68	61.91	63.75
19	27.20	30.14	32.85	36.19	38.58	43.82	50.80	57.37	63.68	65.53
20	28.41	31.41	34.17	37.57	40.00	45.31	52.39	59.04	65.42	67.30
21	29.62	32.67	35.48	38.93	41.40	46.80	53.96	60.70	67.15	69.04
22	30.81	33.92	36.78	40.29	42.80	48.27	55.52	62.34	68.86	70.77
23	32.01	35.17	38.08	41.64	44.18	49.73	57.07	63.97	70.55	72.48
24	33.20	36.42	39.36	42.98	45.56	51.18	58.61	65.58	72.23	74.18
25	34.38	37.65	40.65	44.31	46.93	52.62	60.14	67.18	73.89	75.86
26	35.56	38.89	41.92	45.64	48.29	54.05	61.66	68.77	75.55	77.54
27	36.74	40.11	43.19	46.96	49.64	55.48	63.16	70.35	77.19	79.19
28	37.92	41.34	44.46	48.28	50.99	56.89	64.66	71.92	78.82	80.84
29	39.09	42.56	45.72	49.59	52.34	58.30	66.15	73.47	80.44	82.48
30	40.26	43.77	46.98	50.89	53.67	59.70	67.63	75.02	82.04	84.10
35	46.06	49.80	53.20	57.34	60.27	66.62	74.93	82.64	89.95	92.08
40	51.81	55.76	59.34	63.69	66.77	73.40	82.06	90.08	97.65	99.86
50	63.17	67.50	71.42	76.15	79.49	86.66	95.97	104.54	112.61	114.96
100	118.50	124.34	129.56	135.81	140.17	149.45	161.32	172.10	182.13	185.03
150	172.58	179.58	185.80	193.21	198.36	209.26	223.11	235.60	247.15	250.48
200	226.02	233.99	241.06	249.45	255.26	267.54	283.06	297.00	309.84	313.53
224	251.52	259.91	267.35	276.16	282.27	295.14	311.39	325.96	339.36	343.22
250	279.05	287.88	295.69	304.94	311.35	324.83	341.83	357.04	371.02	375.04
500	540.93	553.13	563.85	576.49	585.21	603.45	626.24	646.48	664.96	670.25

Fisher's F distribution

$F_{.05}$

$P(F \geq F_{.05}) = 0.050$

ν_1

ν_2	1	2	3	4	5	6	7	8	9	10	11	12
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.0	243.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	2.11	2.08
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	2.10	2.07
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	2.09	2.06
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	2.08	2.05
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.07	2.04
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80