

Question 1. [10 marks]

Ohio is a key swing state in the U. S. presidential election. No Republican nominee has ever won without winning Ohio’s electoral votes. On October 3, a Quinnipiac poll based on 497 likely voters showed Trump with 47% and Clinton with 42% of the sample in Ohio. On October 5, a Monmouth poll based on 405 likely voters showed Trump with 42% and Clinton with 44% of the sample in Ohio.

- a) Keeping the probability of a Type I error at 0.05, perform a test to determine whether there is a statistically significant difference **in the support for Trump** between the two polls. Use the critical value approach.

[5]

Sample	X	N	Sample p	
1	234	497	0.470825	Trump1
2	170	405	0.419753	Trump2

Estimate for difference: 0.0510719
 95% CI for difference: (-0.0140122, 0.116156)
 Test for difference = 0 (vs not = 0): Z = 1.53 P-Value = 0.125

Sample	X	N	Sample p	
1	209	497	0.420523	Clinton1
2	178	405	0.439506	Clinton2

Difference = p (1) - p (2)
 Estimate for difference: -0.0189830
 95% CI for difference: (-0.0839450, 0.0459790)
 Test for difference = 0 (vs not = 0): Z = -0.57 P-Value = 0.567

-1 for hypotheses, 1 for pooled proportion of 0.448 (for Trump), 1 for test statistic, 1 for rejection region $|z| > 1.96$, 1 for decision (not to reject H_0) and conclusion of “no difference”.

- b) Find the p-value for the test statistic in (a).

[1] $P(|z| > 1.53) = 2 * 0.063 = 0.126$
 $P(|z| > 0.57) = 2 * 0.2843 = 0.569$

- c) Now calculate the appropriate 95% confidence interval for the difference in popular support for Trump between the two surveys.

[2] $0.05 \pm 1.96 * \sqrt{.47 * .53 / 497 + .42 * .58 / 405} = 0.05 \pm 1.96 * 0.0332 = (-0.015, 0.115)$
 (for Trump), or (for Clinton):
 $0.02 \pm 1.96 * \sqrt{.42 * .58 / 497 + .44 * .56 / 405} = 0.02 \pm 1.96 * 0.0331 = (-0.045, 0.085)$

- d) Explain whether and how the three approaches above are or are not consistent.

[2] p-value not < 0.05 and CI covers zero, therefore do not reject H_0 ; therefore all three approaches are consistent. (Many stated incorrectly that the CI should cover α or the p-value.)

Question 2. [10 marks]

Appendix A shows, for a sample of Ottawa-Nepean homes, both the 2015 and the 2016 assessed values in thousands of dollars. Suppose you are interested in determining whether the typical increase in assessed values is more than \$18,000.

a) Explain whether you should do an independent samples test or a paired samples test.

[1] There is only one sample of homes. Must explain why the samples are dependent (because the assessed values are for the **same homes**). Paired samples test is appropriate.

b) Given your answer to (a), identify the most appropriate test. Explain briefly, with reference to specific boxplots.

[2] For paired sample test, only the differences matter. The differences boxplot suggests it is reasonable to assume a normal distribution of differences. Therefore, the parametric matched t-test is appropriate.

If samples are considered independent in (a), then look at the two samples. Since it is not reasonable to assume these samples come from normally distributed populations, the non-parametric Mann-Whitney is appropriate.

c) Notwithstanding your answer to (b), but being consistent to your answer to (a),

i. Perform the appropriate parametric test to determine whether there is sufficient evidence to conclude that the mean increase is more than \$18,000. Use the 0.05 level of significance.

[4] $H_0: \mu_d = 18$; $H_a: \mu_d > 18$

$$T = (20.73 - 18) / (4.45 / \sqrt{15}) = 2.73 / 1.15 = 2.37$$

Rejection region is one-sided: $t > 1.76$ (prob of 0.05 and 14 df), **not $|t| > 1.76$** .

We reject the null H and conclude the mean increase exceeds \$18,000.

ii. Now perform an appropriate non-parametric test to determine whether there is sufficient evidence to conclude that the median increase is more than \$18,000. Use the 0.05 level of significance.

[3] H_0 : **pop med diff** = 18; H_a : **pop med diff** > 18

$$p\text{-value} = 0.021 < 0.05$$

reject H_0 and conclude the **median** increase exceeds \$18,000

Question 3. [10 marks]

Appendix B shows some graphs and summaries of median household incomes for two samples of neighbourhoods, one from Calgary (HhldInc_Cal) and the other from Edmonton (HhldInc_Edm).

- a) Test at the 0.05 level of significance to determine if there is evidence of a difference in average household income between the two cities. Use a parametric test and do not assume equal population variances.

[5] -Ho: $\mu_1 - \mu_2 = 0$; Ha: $\mu_1 - \mu_2 \neq 0$

-t = $(90716 - 80444) / SE$, where $SE = \sqrt{26715^2/59 + 20688^2/60} = 4385.166$, so

$$t = 10273 / 4385 = 2.34 \quad (2 \text{ marks})$$

-rejection region is $|t| > 1.96$

-Reject Ho, conclude there is evidence of a difference.

- b) Find the p-value for the result above.

[1] p-value is $P(|t| > 2.34) \cong P(|z| > 2.34) = 2 * P(z < -2.34) = 2 * 0.0096 = 0.0192$.
Using t-dist with 109 (or 100) df, p-value very close to $2 * 0.01 = 0.02$,
since $t(0.01) = 2.36$ for 100 df.

- c) Now calculate an appropriate 95% confidence interval for the difference in average household income between the two cities.

[2] $10273 \pm 1.96 * 4385 = 10273 \pm 8595 = (1678, 18868)$

More accurately, $10273 \pm 1.98 * 4385 = 10273 \pm 8682 = (1591, 18955)$

- d) What assumptions regarding the population data are *required* to justify your calculations above? Looking at the boxplots of the two samples of median incomes, explain whether these assumptions are reasonable.

Necessary assumption(s): since both samples are large (> 30), we only need to assume the populations are not extremely skewed (we require the data to be normally distributed only if the samples are small).

- [2] Clearly some skewness exists particularly in the Calgary sample, but it is reasonable to assume the populations are not extremely skewed.

Some solutions stated “boxplots not too skewed” as an assumption. This is an observation, not an assumption regarding the population data. The observation makes the assumption reasonable.

Question 4. [10 marks]

A CPA with a large accounting firm wants to select a random sample of transactions to determine if the population deviation rate (the proportion with material errors) is below the tolerable rate of 7%.

- a) What hypotheses should he test to be able to conclude that the population deviation rate is below 7%? (This means that the control procedures are operating effectively.)

[1] $H_0: p = 0.07$; $H_a: p < 0.07$

- b) Based on his previous auditing experience, he expects to find a deviation rate of 4%. What sample size should he select if he wants to calculate the appropriate 95% confidence interval that will allow him to reject the null hypothesis?

$$\begin{array}{ll} [3] & 0.04 * 0.96 * (1.645/0.03)^2 = 116 & 0.07 * 0.93 * (1.645/0.03)^2 = 196 \\ & 0.04 * 0.96 * (1.96/0.03)^2 = 164 & 0.07 * 0.93 * (1.96/0.03)^2 = 278 \end{array}$$

-1 mark for identifying ME of 0.03 (0.07 – 0.04)

(To reject H_0 , the CI must have an UB of $0.04 + ME < 0.07$. Thus, $ME < 0.07 - 0.04$.)

-1 mark for identifying 1.645 for a 1-sided CI, or 1.96 for a 2-sided CI

-1 mark for using either 0.04 or 0.07 as the value of the proportion.

- c) In your calculations above, you are assuming that the sample proportion has a normal distribution.

(A binomial population or a binomial sample cannot have a normal distribution, and a population proportion is a fixed value and cannot have a distribution.)

Assuming the null hypothesis is true, this is reasonable since

$$np = 196 * .07 > 10, np = 278 * 0.07 > 10,$$

$$\text{or } \underline{\text{not}} \text{ reasonable since } np = 115 * .04 < 10, np = 164 * 0.04 < 10.$$

- d) Suppose he ends up taking a sample size of 200 and finds only 6 transactions with errors. Find the **exact** p-value of this result and state your final conclusion for the test in (a).

[2] the **exact** p-value is $P(X \leq 6) = 0.0119 < 0.05$, based on $n=200$ and $p=0.07$.

Since the p-value < 0.05 , we reject H_0 . The control procedures are operating effectively.

Since $np = 200 * 0.07 > 10$, the **approximate** p-value is based on

$$z = (0.03 - 0.07) / \sqrt{0.07 * 0.93 / 200} = -.04 / 0.018 = -2.22. \text{ Here,}$$

$$p\text{-value} = P(z < -2.22) = 0.0132. \text{ This leads to the same conclusion as above.}$$

This approximate solution is only worth 1 mark.