Similar matrices.

Def: $A \sim B$ if $\exists$ invertible P
st $A = PBP^{-1}$

Remark: If $A = PBP^{-1}$

$$P^{-1}AP = P^{-1}(PBP^{-1})P$$
$$= IBI$$
$$= B$$

Thm

If $A \sim B$ they have <u>same</u> eigenvalues

Proof:
$$|A - \lambda I| = |PBP^{-1} - \lambda I|$$
$$= |PBP^{-1} - \lambda PIP^{-1}|$$
$$= |PBP^{-1} - \lambda (IP^{-1})|$$
$$= |P(B - \lambda I)P^{-1}|$$
$$= |P| \; |B - \lambda I| \; |P^{-1}|$$
$$= |P| \; |B - \lambda I| \; \frac{1}{|P|}$$
$$= |B - \lambda I|$$

$PIP^{-1} = PP^{-1} = I$

5.3 Diagnolization

$\sim$: is similar to.

Def: A is Diagnolization if $A \sim D = \begin{bmatrix} * & & 0 \\ & \ddots & \\ 0 & & * \end{bmatrix}$

$$\begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0.92 \end{bmatrix}$$ ✓ Diagnolizable