CARLETON UNIVERSITY

FINAL EXAMINATION
MATH 1004 A, B, C, D, E, F
December 2013

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004 A, B, C, D, E, F.
Course Instructor(s): Dr. A.B. Mingarelli (Sect. A), Dr. Y. Gao (Sect. B), Mr. M. Blenkinsop (Sect. C, D), Dr. B. Brimacombe (Sect. E), Dr. Z. Montazeri (Sect. F).

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3. **The examination consists of two sheets of legal size paper.** It is out of a total of 100 and consists of 25 multiple choice questions each worth 4 marks **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.,* you get 0 marks for that question).

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1. [**4 marks**] Let $f(x) = |x-1| + |x-3|$. Calculate $L = \lim_{h \to 0^+} \dfrac{f(1+h) - f(3)}{h}$.

   (a) $L = 0$ **(b)** $L = 1$ **(c)** $L = -1$ **(d)** This limit does not exist

2. [**4 marks**] Let $f(x) = \dfrac{1-\cos x}{x^2}$, for $x \neq 0$, and $f(x) = A$, for $x = 0$. What value of A will make f continuous at $x = 0$?

   (a) $A = 0$ **(b)** $A = 1/2$ **(c)** $A = -1$ **(d)** $A = 1$.

3. [**4 marks**] Evaluate $L = \lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x^2 - 9}$.

   (a) $L = 0$ **(b)** $L = \frac{3}{2}$ **(c)** $L = \frac{2}{3}$ **(d)** This limit does not exist

4. [**4 marks**] Let $f(x) = \dfrac{\sin(3x)}{\sin(2x)}$. Evaluate $L = \lim_{x \to 0} f(x)$.

   (a) $L = 0$ **(b)** $L = \frac{3}{2}$ **(c)** $L = \frac{2}{3}$ **(d)** This limit does not exist

5. [**4 marks**] Two functions f, g are defined by $f(x) = 3x^2$ and $g(x) = \cos x$.
   What is the value of their composition $f(g(0))$?

   (a) -3 **(b)** 3 **(c)** -3.2 **(d)** 0

6. **[4 marks]** Find the derivative of the function $y = \dfrac{8x}{\ln(5x+1)}$.

(a) $\dfrac{8(5x+1)\ln(5x+1) - 40x}{(5x+1)(\ln(5x+1))^2}$ **(b)** $\dfrac{1}{\ln(40x)}$ **(c)** $\dfrac{8\ln(5x+1) - 40x}{\ln(5x+1)\,(5x+1)^2}$ **(d)** $\dfrac{8}{5\ln(5x+1)}$

7. **[4 marks]** Find the derivative of the function $y = 5x^2e^{3x}$.

(a) $10xe^{3x}(2x+3)$ **(b)** $5xe^{3x}(2x+3)$ **(c)** $10ex^{3x}(3x+2)$ **(d)** $5xe^{3x}(3x+2)$

8. **[4 marks]** Find the derivative of the function $y = \ln(e^{x^2}+1)$.

(a) $\dfrac{2xe^{x^2}}{e^{x^2}+1}$ **(b)** $\dfrac{2x}{e^{x^2}}$ **(c)** $\dfrac{2xe^{x^2}}{\ln(e^{x^2}+1)}$ **(d)** $\dfrac{2e^{x^2}}{(e^{x^2}+1)^2}$

9. **[4 marks]** Find any local maximum or minimum points of the given function. $y = x^3 - 3x^2 + 1$.

(a) Minimum at $(0,1)$, maximum at $(2,-3)$ **(b)** Maximum at $(0,1)$, minimum at $(2,-3)$ **(c)** Maxima at $(-2,-19)$ and $(0,1)$, minimum at $(2,-3)$ **(d)** Minimum at $(2,-3)$

10. **[4 marks]** Which of the following statements is true?

(a) $f(x) = 2e^x$ is concave down for all x, and has no points of inflection.
(b) $f(x) = x^5 + 1$ is concave up for all x, and has no points of inflection.
(c) $f(x) = x^2 + 5$ is concave up for $x < 0$, concave down for $x > 0$, and has a point of inflection at $(0,5)$.
(d) $f(x) = (x-5)^3$ is concave down for $x < 5$, concave up for $x > 5$, and has a point of inflection at $(5,0)$.

11. **[4 marks]** Evaluate $\displaystyle\int \frac{\sec^2(\ln x)}{x}\,dx$.

(a) $\tan(\ln x) + C$ **(b)** $\ln(\sec x) + C$, **(c)** $2\sec(\ln x) + C$ **(d)** $\ln(\tan x) + C$

12. **[4 marks]** Evaluate the definite integral $\displaystyle\int_0^{\pi} \sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\,dx$

(a) $\dfrac{\pi}{6}$ **(b)** $\dfrac{\pi}{2}$ **(c)** $\dfrac{\pi}{4}$ **(d)** $\dfrac{\pi}{8}$

13. **[4 marks]** Evaluate $I = \displaystyle\int e^{4x}\cos\left(\frac{x}{2}\right)\,dx$.

(a) $\dfrac{1}{12}e^{4x}\left(3\sin\left(\frac{x}{2}\right) + 14\cos\left(\frac{x}{2}\right)\right) + C$ **(b)** $\dfrac{1}{23}e^{4x}\left(2\sin\left(\frac{x}{2}\right) + 3\cos\left(\frac{x}{2}\right)\right) + C$
(c) $\dfrac{1}{65}e^{4x}\left(2\sin\left(\frac{x}{2}\right) + 16\cos\left(\frac{x}{2}\right)\right) + C$ **(d)** $\dfrac{1}{5}e^{4x}\left(\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)\right) + C$

14. **[4 marks]** Evaluate the definite integral $\displaystyle\int_0^3 e^{x/3}(x^2+2x)\,dx$.

(a) 0 **(b)** $27e - 36$ **(c)** $e - 1$ **(d)** $21e + 40$

15. **[4 marks]** Evaluate the definite integral $\displaystyle\int_1^e (x\ln x)^2\,dx$.

(a) $\dfrac{e}{4}$ **(b)** $\dfrac{e^3-1}{2}$ **(c)** $\dfrac{5e^3-2}{27}$ **(d)** $\dfrac{e^2+1}{6}$

16. **[4 marks]** Evaluate $I = \displaystyle\int \frac{4}{x^4-1}\,dx$.

(a) $\ln|x-1| - \ln|x+1| - 2\tan^{-1}(x) + C$ **(b)** $\ln|x^2+1| + 2\tan^{-1}(x) + C$
(c) $\ln|x-1| - 4\ln|x+1| - 2\tan^{-1}(x) + C$ **(d)** $2\ln|x-1| + \ln|x+1| + \tan^{-1}(x) + C$

17. **[4 marks]** Let $f(x) = \sin(\sin 3x)$. Evaluate $f'(\pi/2)$. In other words, find the derivative of f at $x = \pi/2$.

(a) $f'(\pi/2) = 0$ **(b)** $f'(\pi/2) = 1$ **(c)** $f'(\pi/2) = 2$ **(d)** $f'(\pi/2) = 3$

18. [**4 marks**] Evaluate the following limit: $L = \lim_{x \to 0} \frac{\arcsin(5x)}{x^2}$

(a) $L = 5$ **(b)** $L = \frac{1}{5}$ **(c)** $L = 0$ **(d)** This limit does not exist

19. [**4 marks**]. Given that f is such that its inverse F exists, $f'(-5) = 4$, $F(2) = -5$, find the value of the derivative of F at $x = 2$.

(a) 4 **(b)** 1/4 **(c)** 5 **(d)** 1/5

20. [**4 marks**] Let y be given implicity as a differentiable function of x by $2x = xy + y^2$. Then the slope of the tangent line to the curve $y = y(x)$ at the point (x, y) where $x = 1, y = 1$ is equal to:

(a) 2 **(b)** 1/2 **(c)** 3 **(d)** 1/3

21. [**4 marks**] Let $f(x) = 2|x - 5|$. Calculate $L = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$.

(a) $L = 0$ **(b)** $L = 5$ **(c)** $L = -5$ **(d)** This limit does not exist.

22. [**4 marks**] Let $f(x) = \sqrt{x^2 + 4}$. Evaluate $f''(0)$. In other words, find the second derivative of f at $x = 0$.

(a) $f''(0) = 4$ **(b)** $f''(0) = 0$ **(c)** $f''(0) = 1/2$ **(d)** $f''(0)$ does not exist

23. [**4 marks**] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \cos x$ between $x = 0$ and $x = \pi/2$ about the $y-$axis.

(a) $\pi \int_0^{\pi/2} x^2 \cos x \, dx$ **(b)** $\int_0^{\pi/2} \cos x \, dx$ **(c)** $\int_0^{\pi/2} x \sin x \, dx$ **(d)** $2\pi \int_0^{\pi/2} x \cos x \, dx$

24. [**4 marks**] Evaluate the improper integral $\int_0^{\infty} 3x^2 e^{-x} \, dx$.

(a) 12 **(b)** 2 **(c)** 6 **(d)** 1

25. [**4 marks**] Find the area of the region bounded by the curves $y = x^2 + 1$ and $y = 5$.

(a) 16 **(b)** $\frac{8}{3}$ **(c)** $\frac{32}{3}$ **(d)** $\frac{3}{2}$

[Total: 100 marks]

END OF THE EXAMINATION.