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GNG1105/1505
Final Exam

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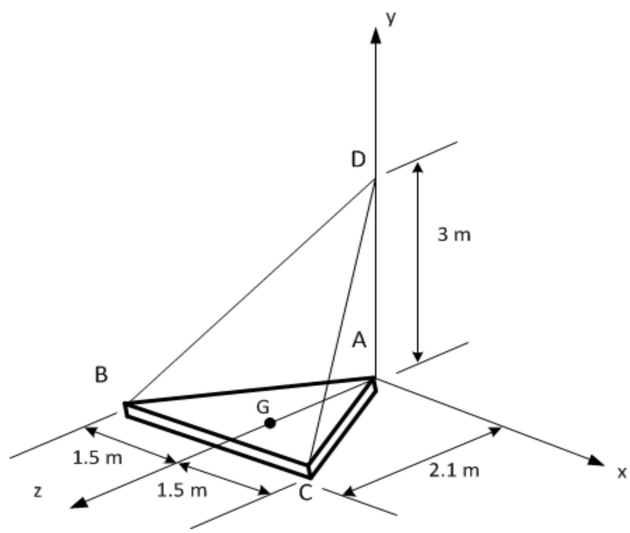
Closed book Examination. Only non- programmable calculators are allowed.
All other electronic devices are not allowed.

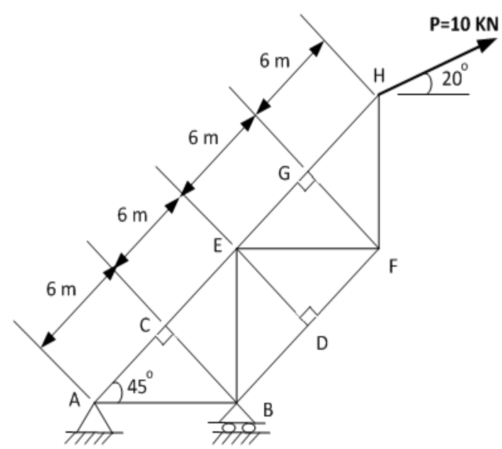
1- The triangular horizontal plate ABC (AB=AC) is being held in by a ball and socket joint at A and two cables (BD and CD), as shown. The weight of the plate is 500N which is acting at its centroid G.

a) Draw the free body diagram of the plate,
b) Write, in vector form, the tension in cables BD,CD, and the 500N weight,
c) Calculate the tensions in cables BD and CD and the components of the reaction at point A.



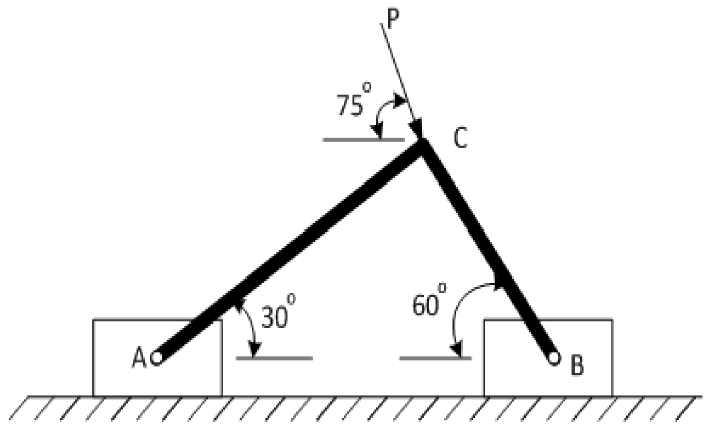
2- The sketch shows a pin-jointed truss loaded with a single force P=10 KN, as shown.

a) Identify all zero-force members,
b) Find the reactions at supports A and B,
c) Find the forces in members AB and AC by the "joints method", and identify if they are in tension or compression,
d) Find the forces in members EG, EF, and DF by the "sections method" and identify if they are in tension or compression.



3- Block A weighs 20 Kg and block B weighs 10Kg. Both are pin-connected by two slender rods of negligible weight at point C, as shown in the diagram.

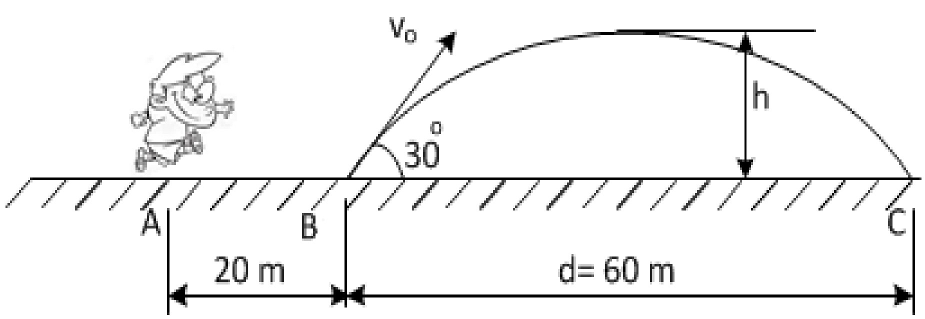
If the coefficient of static friction between the two blocks and the horizontal surface is 0.20, calculate the largest value of P for equilibrium to be maintained.



4- A football player ran from point A to point B, 20 m away with maximum acceleration. He arrived at point B in 2.5 second where he kicked the football with a velocity V_o making 30° angle with the horizontal (as shown in the diagram).

a) Find the acceleration and the velocity of the player at point B.

b) If the distance from point B to point C, where the ball hits the ground, is 60 m, find the value of V_o and the maximum height, h, of the football above the ground.



Useful Equations

$$x = x_o + vt$$
$$v = v_o + at$$
$$x = x_o + v_o t + \frac{1}{2}at^2$$
$$v^2 = v_o^2 + 2a(x - x_o)$$
$$\sum \vec{F} = m\vec{a}$$
$$\sum \overrightarrow{F_x} = m\overrightarrow{a_x} \quad , \quad \sum \overrightarrow{F_y} = m\overrightarrow{a_y} \quad , \quad \sum \overrightarrow{F_z} = m\overrightarrow{a_z}$$

GNG 1105 A & E

ENGINEERING MECHANICS

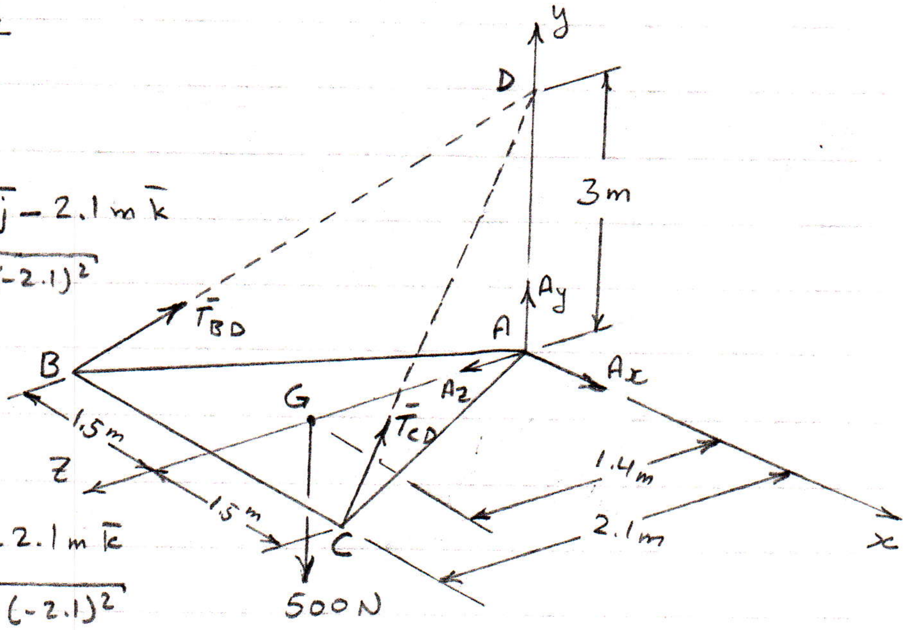
FINAL EXAM

SOLUTIONS

Dec. 12, 2014

1\. a) FBD - plate ABC

See diagram.



b) $\overline{BD} = 1.5\,m\,\bar{i} + 3\,m\,\bar{j} - 2.1\,m\,\bar{k}$

$BD = \sqrt{(1.5)^2 + (3)^2 + (-2.1)^2}$

$= 3.96\,m$

$\overline{CD} = -1.5\,m\,\bar{i} + 3\,m\,\bar{j} - 2.1\,m\,\bar{k}$

$CD = \sqrt{(-1.5)^2 + (3)^2 + (-2.1)^2}$

$= 3.96\,m.$

$$\bar{T}_{BD} = T_{BD}\,\bar{\lambda}_{BD} = T_{BD}\,\frac{\overline{BD}}{BD}$$

$$= \frac{T_{BD}}{3.96}(1.5\bar{i} + 3\bar{j} - 2.1\bar{k}) \qquad \text{—— ANS}$$

$$\bar{T}_{CD} = T_{CD}\,\bar{\lambda}_{CD} = T_{CD}\,\frac{\overline{CD}}{CD}$$

$$= \frac{T_{CD}}{3.96}(-1.5\bar{i} + 3\bar{j} - 2.1\bar{k}) \qquad \text{—— ANS.}$$

$$\bar{W} = -500\,N\,\bar{j} \qquad \text{—— ANS.}$$

c) $BA = CA = \sqrt{(2.1)^2 + (1.5)^2} = \sqrt{6.66} = 2.58\,m.$

$$\Sigma \bar{M}_A = \bar{r}_{B/A}\bar{T}_{BD} + \bar{r}_{C/A}\bar{T}_{CD} + \bar{r}_{G/A}\bar{W} = 0$$

where $\bar{r}_{B/A} = -1.5\bar{i} + 2.1\bar{k}$, $\bar{r}_{C/A} = 1.5\bar{i} + 2.1\bar{k}$; $\bar{r}_{G/A} = 1.4\bar{k}$

(Cont'd on next page)

1. c) Cont'd.

$$\Sigma \bar{M}_A = (-1.5\bar{i} + 2.1\bar{k}) \times \frac{T_{BD}}{3.96}(1.5\bar{i} + 3.0\bar{j} - 2.1\bar{k})$$

$$+ (1.5\bar{i} + 2.1\bar{k}) \times \frac{T_{CD}}{3.96}(-1.5\bar{i} + 3.0\bar{j} - 2.1\bar{k}) + 1.4\bar{k} \times (-500N)\bar{j} = 0$$

$$\Sigma \bar{M}_A = -1.14 T_{BD}\bar{k} - \cancel{0.80 T_{BD}\bar{j}} + \cancel{0.80 T_{BD}\bar{j}} - 1.59 T_{BD}\bar{i}$$

$$+ 1.14 T_{CD}\bar{k} + \cancel{0.80 T_{CD}\bar{j}} - \cancel{0.80 T_{CD}\bar{j}} - 1.59 T_{CD}\bar{i} \quad \bar{k}\ \bar{j}\ \bar{i}$$

$$+ 700\bar{i} = 0$$

$$\therefore \Sigma \bar{M}_A = -1.14 T_{BD}\bar{k} - 1.59 T_{BD}\bar{i} + 1.14 T_{CD}\bar{k} - 1.59 T_{CD}\bar{i} + 700\bar{i} = 0$$

Equate Coefficient of $\bar{i}$ & $\bar{k}$ to zero:

$\bar{i}$: $-1.59\,T_{BD} - 1.59\,T_{CD} + 700 = 0$ ——— (1)

$\bar{k}$: $-1.14\,T_{BD} + 1.14\,T_{CD} = 0$ ——— (2)

$\therefore$ From eq. (2): $T_{BD} = T_{CD}$ which is obvious due to Symmetry

$\therefore$ eq (1): $-1.59 T_{BD} - 1.59 T_{BD} + 700 = 0$

$$3.18\,T_{BD} = 700$$

Hence, $T_{BD} = T_{CD} = \dfrac{700}{3.18} = \underline{\underline{220.13N}} = \underline{\underline{220N}}$ ANS.

$\Sigma F_x = 0$

$$A_x + \frac{T_{BD}}{3.96} \times 1.5 - \frac{T_{CD}}{3.96} \times 1.5 = 0$$

But, $T_{BD} = T_{CD}$,

$\therefore$ $\underline{\underline{A_x = 0}}$ ANS.

$$\Sigma F_y = 0 = A_y + \frac{T_{BD}}{3.96} \times 3.0 + \frac{T_{CD}}{3.96} \times 3.0 - 500N$$

$$\therefore A_y + \frac{220.13}{3.96} \times 3.0 + \frac{220.13}{3.96} \times 3.0 - 500N = 0$$

Hence, $\underline{\underline{A_y}} = -166.77 - 166.77 + 500N = \underline{\underline{166.46N}}$ ANS.

$$\Sigma F_z = 0 = A_z - \frac{T_{BD}}{3.96} \times 2.1 - \frac{T_{CD}}{3.96} \times 2.1$$

$$\therefore A_z - \frac{220.13}{3.96} \times 2.1 - \frac{220.13}{3.96} \times 2.1 = 0$$

Hence, $\underline{\underline{A_z}} = 116.74 + 116.74 = \underline{\underline{233.48N}}$ ANS.

1. c) <u>Another Method</u>

$$\Sigma \bar{M}_A = \bar{r}_{B/A} \bar{T}_{BD} + \bar{r}_{C/A} \bar{T}_{CD} + \bar{r}_{G/A} \bar{W} = 0$$

$$= T_{BD} \times \begin{array}{ccc} (+) & (-) & (+) \\ \end{array} \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1.5 & 0 & 2.1 \\ 0.38 & 0.76 & -0.53 \end{vmatrix} + T_{CD} \times \begin{array}{ccc} (+) & (-) & (+) \\ \end{array} \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1.5 & 0 & 2.1 \\ -0.38 & 0.76 & -0.53 \end{vmatrix} + \begin{array}{ccc} (+) & (-) & (+) \\ \end{array} \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & 1.4 \\ 0 & -500 & 0 \end{vmatrix} = 0$$

$$= -2.1 \times 0.76\, T_{BD} \bar{i} - [-1.5 \times (-0.53) - 2.1 \times 0.38]\, T_{BD} \bar{j} - 1.5 \times 0.76\, T_{BD} \bar{k}$$
$$- 2.1 \times 0.76\, T_{CD} \bar{i} - [1.5 \times (-0.53) - 2.1 \times (-0.38)]\, T_{CD} \bar{j} + 1.5 \times 0.76\, T_{CD} \bar{k}$$
$$- 1.4 \times (-500) \bar{i} = 0$$

$$= -1.60 T_{BD} \bar{i} - 0.0 T_{BD} \bar{j} - 1.14 T_{BD} \bar{k}$$
$$- 1.60 T_{CD} \bar{i} - 0.0 T_{CD} \bar{j} + 1.14 T_{CD} \bar{k} + 700 \bar{i} = 0$$

$$= -1.60 T_{BD} \bar{i} - 1.6 T_{CD} \bar{i} + 700 \bar{i} - 1.14 T_{BD} \bar{k} + 1.14 T_{CD} \bar{k} = 0$$

Now, equate coefficients of $\bar{i}$ and $\bar{j}$ to zero:

$\bar{i}$: $-1.60\, T_{BD} - 1.6\, T_{CD} + 700 = 0$ ——— (1)

$\bar{k}$: $-1.14\, T_{BD} + 1.14\, T_{CD} = 0$ ——— (2)

From (2): $T_{BD} = T_{CD}$

Insert in (1):

$$-1.60 T_{BD} - 1.6 T_{BD} + 700 = 0$$

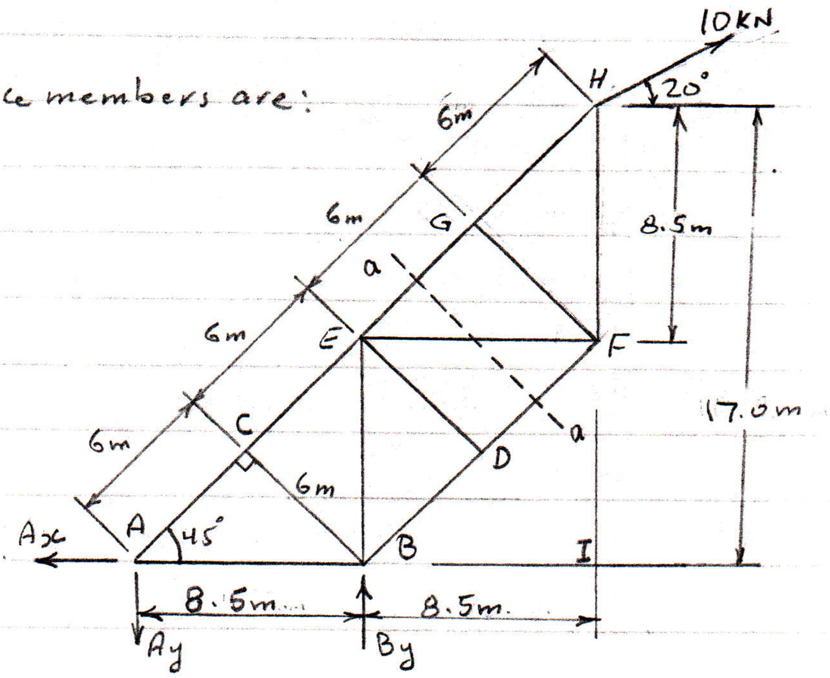
$$3.2\, T_{BD} = 700$$

$$\therefore T_{BD} = T_{CD} = \frac{700}{3.2} = \underline{\underline{218.75\,N}} = \underline{\underline{219\,N}} \simeq \underline{\underline{220\,N}}$$ AN

2. a)

By inspection, zero-force members are:

BC, DE and GF



b) FBD – Entire truss

$$AB = HF = \frac{6m}{\cos 45^\circ} = 8.5m$$

$BI = IF = 12\cos 45^\circ = 8.5m.$

$\therefore HI = 17.0\,m.$

$+\circlearrowleft \Sigma M_A = 0$

$$B_y \times 8.5m - 10KN\cos 20^\circ \times 17.0m + 10KN \sin 20^\circ \times 17.0m = 0$$

$$8.5B_y - 159.7 + 58.1 = 0$$

$$8.5B_y = 101.6, \quad \therefore B_y = \frac{101.6}{8.5} = \underline{\underline{11.95KN\uparrow}}$$ ANS.

$+\uparrow \Sigma F_y = 0$

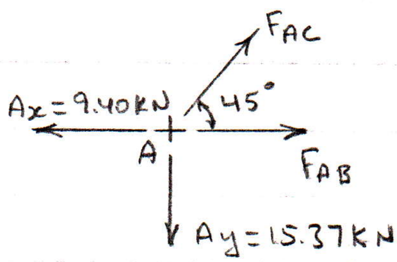
$$B_y - A_y + 10KN \sin 20^\circ = 0$$

$$11.95 - A_y + 3.42 = 0, \quad \therefore A_y = \underline{\underline{15.37KN\downarrow}}$$ ANS.

$\xrightarrow{+} \Sigma F_x = 0$

$$-A_x + 10KN\cos 20^\circ = 0, \quad \therefore A_x = \underline{\underline{9.40KN}} \leftarrow$$ ANS.

c) FBD – Joint A



$+\uparrow \Sigma F_y = 0; \; F_{AC}\sin 45^\circ - A_y = 0$

$$0.707F_{AC} = A_y = 15.37$$

$$\therefore F_{AC} = \frac{15.37}{0.707} = \underline{\underline{21.74KN(T)}}$$ ANS.

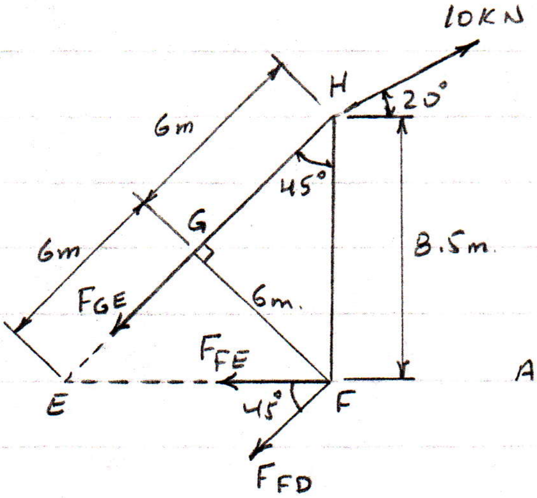
$\xrightarrow{+} \Sigma F_x = 0; \; F_{AC}\cos 45^\circ + F_{AB} - A_x = 0$

$$21.74 \times 0.707 + F_{AB} - 9.40 = 0$$

$$\therefore F_{AB} = \underline{\underline{5.97KN(T)}}$$ ANS.

2.(Cont'd)

d) FBD - Right of Sec. a-a



+↺ $\Sigma M_F = 0$

$F_{GE} \times 6m - 10KN \cos 20° \times 8.5m = 0$

$6 F_{GE} = 79.87$

$\therefore F_{GE} = \frac{79.87}{6} = \underline{\underline{13.3 \, KN \, (T)}}$ ANS.

+↺ $\Sigma M_E = 0$

$-F_{FD} \times 6m - 10KN \cos 20° \times 8.5m + 10KN \sin 20° \times 8.5m = 0$

$-6 F_{FD} = 79.87 - 29.07 = 50.8$

$\therefore F_{FD} = \frac{-50.8}{6} = \underline{\underline{8.5 \, KN (C)}}$ ANS.

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{FE} + F_{FD} \cos 45° - F_{GE} \cos 45° + 10 KN \cos 20° = 0$

$-F_{FE} + 8.5 \cos 45° - 13.3 \cos 45° + 10 \cos 20° = 0$

$F_{FE} = +6.01 - 9.40 + 9.40$

$\therefore F_{FE} = +6.01 KN = \underline{\underline{6.01 KN (T)}}$ ANS.

3.

Block A $= 20\,kg \times 9.81 = 196.2\,N$

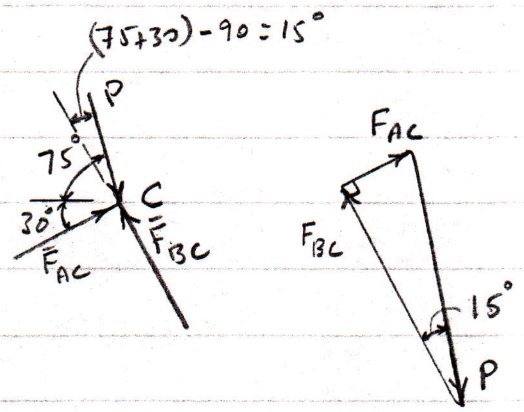
Block B $= 10\,kg \times 9.81 = 98.1\,N$

FBD - Joint C

Look at the Force triangle to the right:

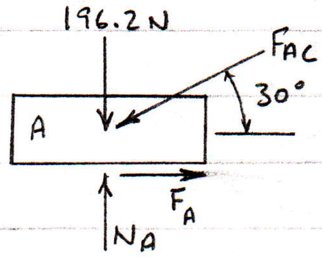
$F_{AC} = P \sin 15^\circ = 0.2588\,P$

$F_{BC} = P \cos 15^\circ = 0.9659\,P$



FBD - Block A

Assume that motion impending will start at block A first.



$+\uparrow \Sigma F_y = 0; \quad N_A - 196.2 - F_{AC} \sin 30^\circ = 0$

$N_A = 196.2 + 0.2588\,P \sin 30^\circ$

$N_A = 196.2 + 0.1294\,P$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_A - F_{AC} \cos 30^\circ = 0$

$F_A = 0.2588\,P \cos 30^\circ = 0.2241\,P$

For motion impending at A: $F_A = \mu_s N_A$

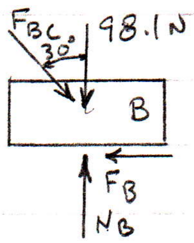
i.e. $N_A = \dfrac{F_A}{\mu_s} = \dfrac{0.2241P}{0.2} = 1.1205\,P$

Insert $N_A = 196.2 + 0.1294P$

$196.2 + 0.1294P = 1.1205\,P$

$0.9911\,P = 196.2 \quad \therefore P = \dfrac{196.2}{0.9911} = \underline{197.96\,N}$

FBD - Block B (Assume that motion impends at B)



$+\uparrow \Sigma F_y = 0; \quad N_B - 98.1 - F_{BC} \cos 30^\circ = 0$

$N_B = 98.1 + 0.9659\,P \cos 30^\circ$

$N_B = 98.1 + 0.8365\,P$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} \sin 30^\circ - F_B = 0$ (Cont'd on next page)

3. (Cont'd)

$F_B = F_{BC} \sin 30° = 0.9659P \sin 30° = 0.4830P$

For motion impending at B:

$F_B = \mu_s N_B$

$N_B = \dfrac{F_B}{\mu_s} = \dfrac{0.4830P}{0.2} = 2.415P$

Insert $N_B = 98.1 + 0.8365P$

$98.1 + 0.8365P = 2.415P$

$1.5785P = 98.1$

$\therefore P = \dfrac{98.1}{1.5785} = 62.15N$

$\therefore$ For equilibrium to be maintained, $\underline{\underline{P = 62.15N}}$ ANS.

4. a)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$20\,m = 0 + 0 + \frac{1}{2} a \times (2.5)^2$$

$\therefore a = \frac{20 \times 2}{(2.5)^2} = \underline{\underline{6.4\ m/s^2}}$ is the acceleration at B. ANS.

$$v = v_0 + at$$

$$v = 0 + 6.4 \times 2.5$$

$\therefore v = \underline{\underline{16\ m/s}}$ is the velocity at point B. ANS.

b)

- Horizontal Motion

$$x = x_0 + (v_0)_x t$$

$$60 = 0 + v_0 \cos 30^\circ t$$

$$v_0 t = \frac{60}{\cos 30^\circ} = 69.28 \quad \text{——— } (1)$$

- Vertical Motion

$$y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$$

$$0 = 0 + v_0 \sin 30^\circ t - \frac{1}{2} \times 9.81 t^2$$

$$4.9 t^2 = 0.5 v_0 t \quad \text{——— } (2)$$

Insert (1) in (2):

$$4.9 t^2 = 0.5 \times 69.28$$

$$t^2 = \frac{0.5 \times 69.28}{4.9} = 7.07$$

$$\therefore t = 2.66\,s.$$

Insert in (1): $2.66 v_0 = 69.28$, $\therefore \underline{\underline{v_0 = 26.05\ m/s}}$ ANS.

$v^2 = v_0^2 + 2a(h - h_0)$ (In the vertical plane)

$$0 = (v_0)_y^2 - 2gh$$

$$0 = (26.05 \sin 30^\circ)^2 - 2 \times 9.81 h$$

$\therefore h = \frac{(26.05 \sin 30^\circ)^2}{2 \times 9.81} = \frac{169.65}{19.62} = \underline{\underline{8.65\,m}}$ ANS.

-END-