ex. Suppose pdf of X is

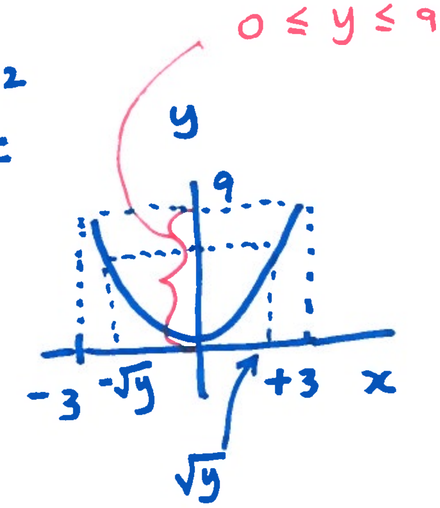
$$\underline{\underline{f(x)}} = \begin{cases} \dfrac{x^2}{18} & \underline{\underline{-3}} < x < 3 \\ 0 & \text{o.w.} \end{cases}$$

$Y = X^2$

$P(Y < 9) = ?$ Solve cdf of $\underline{\underline{Y}}$.

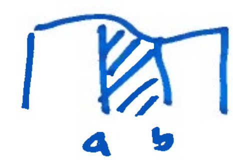
$\underline{\underline{Y = X^2}}$

$\uparrow$



Sol: def. cdf: $F_X(x) = \int_{-\infty}^{x} f(t)\,dt$

$$F(x) = \int_{-3}^{x} \frac{t^2}{18}\,dt = \frac{t^3}{3 \cdot 18}\Bigg|_{-3}^{x} = \underline{\frac{1}{54}\left[x^3 + 27\right]}$$ cdf X



$P(a < X < b)$
$= P(X < b) - P(X < a)$

$$\begin{aligned} G_Y(y) &= P(Y \le y) \\ &= P(X^2 \le y) \\ &= P(-\sqrt{y} \le X \le \sqrt{y}) \\ &= P(X \le \sqrt{y}) - P(X \le -\sqrt{y}) \\ &= F_X(\sqrt{y}) - F_X(-\sqrt{y}) \\ &= \frac{1}{54}\left[y^{3/2} + \cancel{27} - (-y^{3/2} + \cancel{27}) \right] \\ &= \frac{1}{54}\, 2 \cdot y^{3/2} \end{aligned}$$

$$G_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{y^{3/2}}{27} & 0 \le y < 9 \\ 1 & y > 9 \end{cases}$$

$$P(Y < 9) = G_Y(9) = \frac{1}{27} 9^{3/2} = 1$$

WW 4 # 2

components parallel.

$f(x) = \frac{1}{150} e^{-x/150}$ $x > 0$

$X \sim \exp(\lambda = \frac{1}{150})$ lifetime component.

Y = lifetime system = $\max(X_1, X_2, X_3)$

$f(x) = \lambda e^{-\lambda x}$

$F(x) = 1 - e^{-\lambda x}$

$X \sim \exp(\lambda)$

$P(Y \leq y) = P(X_1 \leq y \text{ and } X_2 \leq y \text{ and } X_3 \leq y)$

cdf of Y

$= P(X_1 \leq y) P(X_2 \leq y) P(X_3 \leq y)$

X_i independent.

$= F_{X_1}(y) F_{X_2}(y) F_{X_3}(y)$

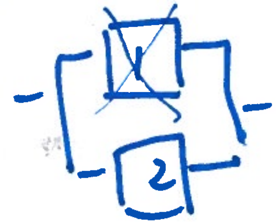
$= (1 - e^{-y/150})^3$

not exponentially dist.

Series

vs.

parallel.



$Y =$ life sys.

fails first.

$Y = \min(X_1, X_2)$

$X_i \sim \exp(\lambda)$

$Y \sim \exp$

$Y = \max(X_1, X_2)$

ex.

$X \sim \exp(\lambda)$

$Y = \min(X_1, X_2)$

what cdf Y?

$$P(Y \le y) = 1 - P(Y > y)$$

$$= 1 - P(\underline{X_1 > y \text{ and } X_2 > y})$$

$$= 1 - P(X_1 > y)\,P(X_2 > y)$$

assume X_i indep.

$$= 1 - (1 - P(X_1 \le y))(1 - P(X_2 \le y))$$

$$= 1 - (1 - F_X(y))^2$$

$$= 1 - (\cancel{1} - (\cancel{1} - e^{-\lambda y}))^2$$

$$= 1 - e^{-\lambda y 2}$$

$Y = \min.$

$Y \sim \exp$

WW4 #4 Q4

$X_1 \sim U(30, 50)$ $\quad f_{X_1}(x) = \frac{1}{b-a} = \frac{1}{50-30} = \frac{1}{20}$ $\quad F_{X_1}(x) = \frac{x-a}{b-a} = \frac{x-30}{20}$

$X_2 \sim U(35, 53)$ $\quad f_{X_2}(x) = \frac{1}{53-35} = \frac{1}{18}$ $\quad F_{X_2}(x) = \frac{x-35}{18}$

Y = project completion time.

$Y = \max(X_1, X_2)$

$$P(Y \le y) = P(X_1 \le y, X_2 \le y)$$

$$= P(X_1 \le y) P(X_2 \le y) \quad \text{independent.}$$

$$= \frac{y-30}{20} \cdot \frac{y-35}{18}$$

$$F_Y(y) = \begin{cases} 0 & y < 35 \\ \frac{y-30}{20} \cdot \frac{y-35}{18} & 35 \le y < 50 \\ \frac{y-35}{18} & 50 \le y < 53 \\ 1 & y \ge 53 \end{cases}$$

30 —— 50

35 —— 53

(30, 35) (35, 50) (50, 53)

↑ not possible to finish before 35h.

$F_Y(y) = 0$

↑ only partner.

a) $$P(Y < 48+4 \mid Y > 48) = \frac{P(Y < 52 \cap Y > 48)}{P(Y > 48)}$$

$$= \frac{F_Y(52) - F_Y(48)}{1 - F_Y(48)} = \frac{\left(\frac{52-35}{53-35}\right) - \left(\frac{48-30}{20} \cdot \frac{48-3}{18}\right)}{1 - \left(\frac{48-30}{20} \cdot \frac{48-35}{18}\right)}$$

$$= 0.8413$$

Problem 4.12 Suppose an enemy aircraft flies directly over the Alaska pipeline and fires a single air-to-surface missile. If the missile hits anywhere within 10 feet of the pipeline, a major structural damage will occur and the oil flow will be disrupted. Let X be the distance from the pipeline to the point of impact. Note that X is a continuous random variable. The probability function describing the missile's point of impact is given by

$$f(x) = \begin{cases} \frac{60+x}{3600} & \text{for } -60 < x < 0 \\ \frac{60-x}{3600} & \text{for } 0 \le x < 60 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the distribution function, $F(x)$.
(b) Let A be the event "flow is disrupted." Find $P(A)$.
(c) Find the mean and the standard deviation of X.
(d) Find the median and the interquartile range of X.

Problem 4.13 Consider a random variable X which follows the uniform distribution on the interval $(0, 1)$. (a) Give the density function $f(x)$ and obtain the cumulative distribution function$F(x)$ of X;
(b) Calculate the mean (expectation) $\mathrm{E}(X)$ and variance $\mathrm{Var}(X)$;
(c) Let $Y = \sqrt{X}$. Find the $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$;
(d) Obtain the distribution function $G(y)$ and furthermore the density function $g(y)$ of random variable Y.

Problem 4.14 The reaction time (in seconds) to a certain stimulus is a continuous random variable with density given below

$$f(x) = \begin{cases} \frac{3}{2x^2} & \text{for } 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Obtain the distribution function.
(b) Take next two observations X_1 and X_2 (we can assume they are *i.i.d*). Then consider $V = \max\{X_1, X_2\}$. What is the density and distribution functions of V?
(c) Compute the expectation $\mathrm{E}(V)$ and the standard deviation $\mathrm{SD}(V)$.
(d) Compute the difference between the expectation and the median for the distribution of V.

4.7.2 Exercise Set B

Problem 4.15 The continuous random variable X takes values between -2 and 2 and its density function is proportional to
(a) $4 - x^2$
(b) x^2
(c) $2 + x$
(d) $\exp\{-|x|\}$

Find, in each case, the density function, the distribution function, the mean, the standard deviation, the median and the interquartile range of X.

problem 4.14

a) $F_X(x) = \int_1^x f(t)\,dt = \int_1^x \frac{3}{2t^2}\,dt$

$= \frac{3}{2}\int t^{-2}\,dt$

$= -\frac{3}{2} t^{-1} \Big|_1^x = -\frac{3}{2}x^{-1} + \frac{3}{2}$

$= \frac{3}{2}\left[1 - \frac{1}{x}\right)$

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{3}{2}\left[1-\frac{1}{x}\right] & 1 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

b) $V = \max(X_1, X_2)$

$F_V(v) = P(V \le v) = P(X_1 \le v \text{ and } X_2 \le v)$

$= P(X_1 \le v)\, P(X_2 \le v)$ assume X_i indep.

$= F_X(v)^2$

$= \left(\frac{3}{2}\left(1 - \frac{1}{v}\right)\right)^2 = \frac{9}{4}\left(1 - \frac{1}{v}\right)^2 = \frac{9}{4} - \frac{2}{2v} + \frac{9}{4v^2}$

$f_v(v) = \frac{d}{dv}\left(\frac{9}{4} - \frac{9}{2}v^{-1} + \frac{9}{4}v^{-2}\right)$

$= 0 + \frac{9}{2}v^{-2} - \frac{9}{4}\cdot 2v^{-3} = \frac{9}{2v^2} - \frac{9}{2v^3}$